

NASA Contractor Report 145244

PLANS - A FINITE ELEMENT PROGRAM FOR NONLINEAR ANALYSIS OF STRUCTURES VOLUME II - USERS' MANUAL

(NASA-CR-145244) PLANS; A FINITE ELEMENT
PROGRAM FOR NONLINEAR ANALYSIS OF
STRUCTURES. VOLUME 2: USER'S MANUAL
(Grumman Aerospace Corp.) 251 p HC A12/MF
A01

N77-33535
Unclas
CSCL 13M G3/39 49407

A. PIFKO, H. ARMEN, JR., A. LEVY, AND H. LEVINE

NASA CONTRACT NAS1-13148

GRUMMAN AEROSPACE CORPORATION

BETHPAGE, NY

MAY 1977



National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U.S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM THE BEST COPY FURNISHED US BY THE SPONSORING AGENCY. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE.

PLANS — A FINITE ELEMENT PROGRAM FOR
NONLINEAR ANALYSIS OF STRUCTURES

VOLUME II — USERS' MANUAL

by

A. Pifko, H. Armen Jr., A. Levy, and H. Levine

Materials and Structural Mechanics

May 1977

Approved by:


Richard A. Scheuing
Director of Research

FOREWORD

The PLANS system of programs for the nonlinear analysis of structures was developed within the Grumman Research Department with the cooperation, guidance, and partial support of the NASA Langley Research Center.

The programs are an outgrowth of the work reported in a series of NASA Contractor Reports: CR-803, CR-1649, CR-2351, and CR-2568. The last-named document is the theoretical companion volume to this manual.

The principal developers of PLANS are Drs. A. Pifko, H. Armen Jr., H. Levine, and A. Levy. The successful development of a system as broad in scope and complexity as PLANS requires the efforts of many individuals. The principal developers gratefully acknowledge the contributions of J. S. Miller for his efforts associated with the initial programming. Special thanks also go to Ms. P. Ogilvie, P. Zirk, and E. Yander for their diligence and dedication to the programming effort.

TABLE OF CONTENTS

<u>Section</u>	<u>Page</u>
1 Introduction and Input Overview	1-1
2 Instructions for the Use of BEND, A Program for the Elastic-Plastic Analysis of Built-Up Structures	2-1
3 Instructions for the Use of HEX, A Program for the Elastic-Plastic Analysis of Three Dimensional Solids	3-1
4 Instructions for the Use of REVBY, A Program for the Elastic-Plastic Analysis of Bodies of Revolution	4-1
5 Instructions for the Use of OUT-OF-PLANE GM, A Program for the Nonlinear Analysis of Built-Up Structures	5-1
6 Instructions for Use of SATELLITE, A Pre- processing Program for PLANS	6-1
7 Example Input	7-1

SECTION 1.

INTRODUCTION AND INPUT OVERVIEW

INTRODUCTION AND INPUT OVERVIEW

The PLANS system, rather than being one comprehensive computer program, is a collection of finite element programs used for the nonlinear analysis of structures. This collection of programs evolved and is based on the organizational philosophy in which classes of analyses are treated individually based on the physical problem class to be analyzed. On the basis of this concept, each of the independent finite element computer programs of PLANS, with an associated element library, can be individually loaded and used to solve the problem class of interest. A number of programs have been developed for material nonlinear behavior alone and for combined geometric and material nonlinear behavior. Table 1 summarizes the usage, capabilities, and element libraries of the current programs of the PLANS system. These include:

- | | |
|-------------------|--|
| • BEND | for the plastic analysis of built-up structures where bending and membrane effects are significant |
| • HEX | for problems requiring a three dimensional elastic-plastic analysis |
| • REVBY | for the plastic analysis of bodies of revolution |
| • OUT-OF-PLANE MG | for the material and geometric nonlinear analysis of built-up structures |

Supplementing these is a SATELLITE program for data debugging and plotting of input geometries.

TABLE 1 OPERATIONAL MODULES OF PLANS - CURRENT STATUS (5/77)

PROGRAM	APPLICATION	ELEMENT LIBRARY	# ELEM	# NODE	TYPE OF LOADING	MATERIAL BEHAVIOR	EQUILIB. CORRECT	SUB-INCREM.	RESTART	PLOTTING	MESH GENERATOR
BEND	Analysis of 3-D thin-walled built-up structures where bending is significant Plastic analysis only	• Beam (9 cross-sections) • Stringer (CSS-LSS) • Triangle Membrane Family (CST-LST) • Triangle Bending & Membrane	• Variable	• Variable	• Surface • Concentrated • Edge • Thermal (Ex. Beams) • Proportional only • Cyclic	• Isotropic-Orthotropic Kinematic Hardening • Linear & Nonlinear Hardening • Ideally Plastic	Yes	No	Yes	Yes (see Satellite)	None
HEX	Analysis of massive 3-D bodies Plastic analysis only	• 3-D Hexahedron 8 node - 20 node Isoparametric	• 2500	• 2500	• Surface • Concentrated • Thermal • Cyclic • Proportional only	• Isotropic Kinematic Hardening • Linear & Nonlinear Hardening • Isotropic-Orthotropic • Ideally Plastic	Yes	No	Yes	None	Limited to 8 node mem.
REVBY	Axisymmetric analysis of bodies of revolution Plastic analysis only	• Isoparametric Shell of Rev. • Rev. Triangle • Thin Ring	• 600	• 900	• Surface • Concentrated • Line • Cyclic • Proportional only	• Same as HEX	No	No	No	Yes (see Satellite)	None
CPLANE-GH	Analysis of 3-D thin-walled built-up structures where bending is not significant Elastic & Geometric Nonlinearity (Small strain-large deflect.)	• Beam (9 cross-sections) • Stringer • Triangle Membrane Family (CST-LST)	• 600	• 900	• Surface • Concentrated • Edge • Thermal (Ex. Beams) • Cyclic • Non-Proportional	• As in BEND	Yes	No	Yes	Yes (see Satellite)	None

In addition, a number of spin-off special purpose modules have been developed: FAST for fracture analysis, AXSHEL for the nonlinear analysis of shells of revolution, and BENST for the buckling analysis of large built-up structures. A general purpose nonlinear dynamic analysis module, DYCAST, is in final stages of development. These modules are not included in this document.

The modules that analyze material nonlinearities alone employ the "initial strain" concept within an incremental procedure to account for the effect of plasticity and include the capability for cyclic plastic analysis. The "initial strain" approach does not require that the stiffness matrix be updated at each step in the analysis but rather, the effect of plasticity enters into the analysis as an effective load vector. The cyclic plastic behavior is accounted for by implementing the Prager-Ziegler kinematic hardening theory. Geometric nonlinearities are included in the PLANS system programs by making use of an "updated" or convected coordinate approach, which requires the reformation of the stiffness matrix due to changes in the geometry and stress field during the incremental procedure.

The theoretical foundations upon which the PLANS system programs are based can be found in a companion volume, "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractor Report NASA CR-2568.

This present volume describes the input data preparation for the four principal modules of PLANS, BEND, HEX, REVBY, and OUT-OF-PLANE MG as well as the SATELLITE program for input checking. In keeping with the philosophy of the PLANS system, the description of the input for each of the modules is in a self-contained section. Also presented is a section that shows the input data decks for a representative number of sample problems.

The input for all the modules begins with a title card that allows for any 80 character title (specified in columns 1-80). This title serves as a page heading for subsequent computer output. The input data following this card is divided into a number of functional groups, each describing a specific type of input information. These groups are briefly described below and schematically shown in Fig. 1. Each input group must be read in the specific order shown in the figure. In general, each group is delineated with a specific section end card. This is the alphanumeric SEND, left justified on the input card, in columns 1 through 4 or a blank card. The input groups are as follows:

- I Title Card
- II Program Control Parameters and Options
- III Node Specification
 This section defines an allowable set of node
 point identification numbers
- IV Element Connectivity
 Defines each element by specifying its type
 (i.e., beam, triangle, ..., etc.), identifica-
 tion number and connecting node points
- V - VII Node Point Coordinates
 Defines the location of each node point in a
 global cartesian coordinate system
- VIII Node Point Single and Multipoint Constraints
 Defines boundary conditions and nodal con-
 straint equations
- IX Element Material and Section Properties
 Section properties include element thickness,
 area and moment of inertia where applicable.

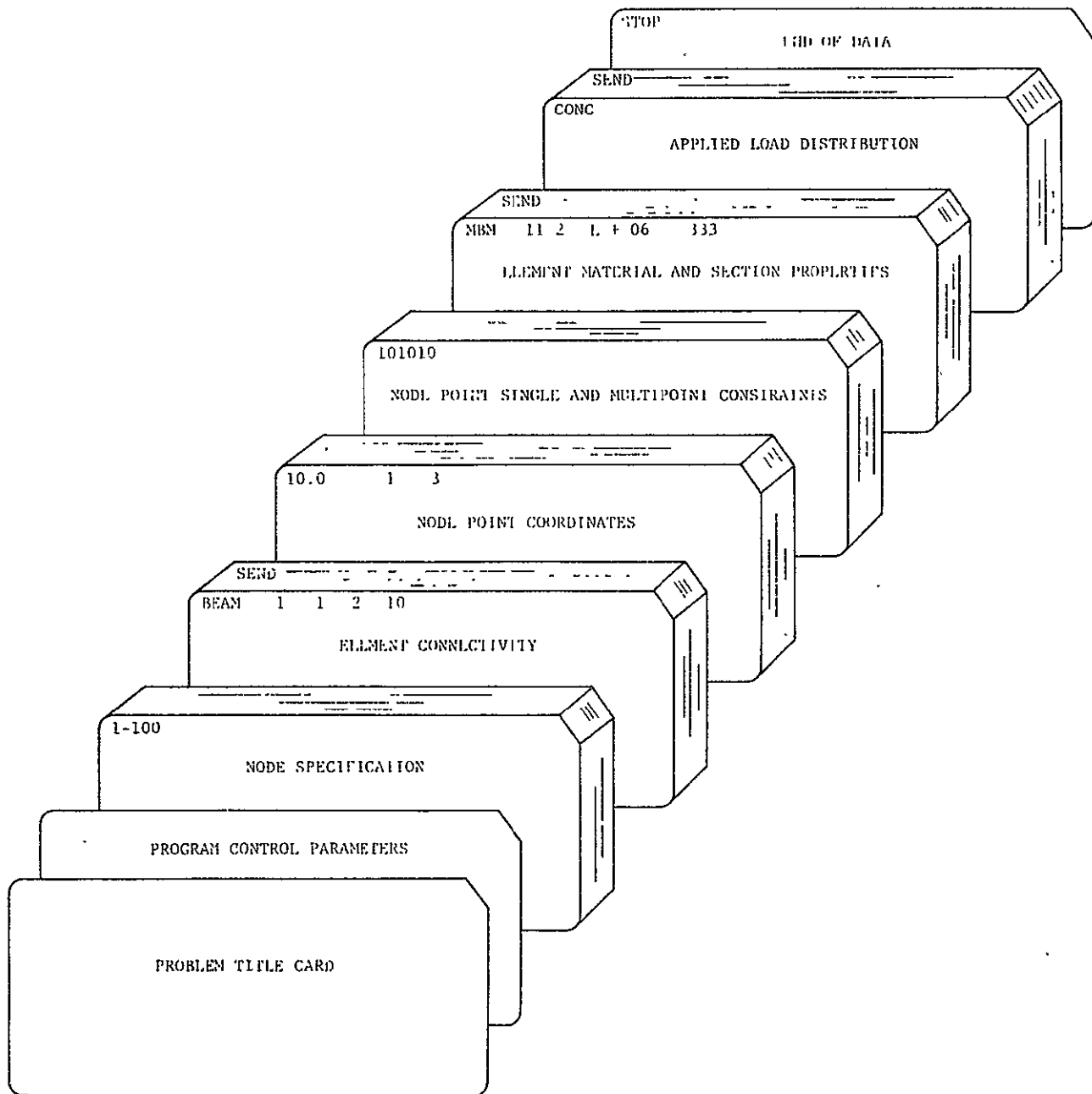


Fig. 1 Order of Groups of Input Data

Material properties include such quantities as Young's modulus, Poisson's ratio, and quantities defining the plastic response such as yield stress and hardening parameters.

X Applied Load

Defines spatial distribution of applied load

The last card in the deck is an alphanumeric STOP or END left justified on an input card in columns 1 through 4. STOP indicates that the job is complete and END indicates another problem deck follows.

Some general rules have been used in designing the input. These rules are listed below:

- Some of the data sections make use of a "keyword" of up to four characters, left justified in their appropriate field to specify an item of data. For example, cards specifying element connectivity for membrane triangles begin with TRIM, and material properties by MAT1. All sections that make use of keywords end with a section end keyword SEND
- Sections that specify data without keywords end with a blank card section delimiter
- The data deck ends with one of two keywords. If END is used, the program reads a new problem data deck. If STOP is used, as will probably be the case for most problems, the job ends
- Two formats for input are used, E15.7 for floating point input (fields of 15) and I5 for integer or fixed point input (fields of 5). The fixed point

(integer) data must be right justified. The floating point data can be written in several forms. For example, 10.0 can be input as:

10.0 any place in the field, or
1.0E+01, 1.0E+1, 1.0E1, where the
field is right justified

There are a number of places in the program where applicable node points or elements must be specified with a set of data. In these cases the nodes or members are specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and -n consecutively is the equivalent of the specification of nodes or elements m, m+1, m+2, ..., n and specifying m, -p, and -n consecutively is the equivalent of the specification of nodes (elements) m, m+p, m+2p, ..., m+kp where m+kp is the highest integer of the form less than or equal to n. For example, the specification of nodes 1 through 100 is written as 1-100 and nodes 1, 3, 5, ..., 99 as 1, -2, -99. This card input appears in fields of 5 (I5 format) with 16 items per card. Any number of continuation cards may be used. A blank I5 field ends the specification.

INSTRUCTIONS FOR USE OF BEND

A Program for the Elastic-Plastic Analysis
of Built-up Structures

INSTRUCTIONS FOR USE OF BEND

BEND is a finite element program to treat the elastic, elastic-plastic or elastic-cyclic plastic response of arbitrary built-up thin walled structures where bending and membrane effects are equally important. The finite element library consists of the following elements:

- Three-node uniform strain triangle
- Six-node linearly varying strain triangle
- Four- and five-node hybrid triangles to be used as transition elements between three- and six-node elements (see Fig. 1a)
- Two-node uniform strain stringer
- Three-node linearly varying strain stringer
- Beam with various cross sections subjected to bending about two planes as well as torsion
- Higher order triangular plate element with bending and membrane capability

The program is capable of treating the elastic and elastic-ideally plastic, linear strain hardening, and nonlinear strain hardening behavior of orthotropic materials. Further, the kinematic hardening theory of plasticity is used (Refs. 1-3) so that provision for cyclic loading conditions involving reversed plastic deformation is included.

The input to the program is categorized in the following sections:

- I. Problem Title FORMAT (20A4)
Any 80-character title describing the problem.
- II. NPNTC, NPRNT, IRESRT, NUTAP, INPRT FORMAT (5I5)
 $0 \leq \text{NPNTC} \leq 63$:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0 No intermediate printout
 = 1 Print the load vector
 = 2 Print element stiffness matrix
 = 16 Print each element stiffness matrix entry
 to be stacked with its stacking index
 = 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, $NPNTC = 1 + 32 = 33$.

NPRNT:

If ≤ 0 , perform elastic analysis only
If > 0 , perform plastic analysis, printing output every
NPRNT increments of load

IRESRT: (See Section XVIII.)

- 0, Elastic (and/or) plastic run. Do not generate a restart tape
- 1, Elastic (and/or) plastic (cyclic) run. Build a new restart tape
- 2, Plastic run with elastic values from previously created restart tape
- 3, Plastic restart run starting at some specified load level
- 4, Cyclic restart run

NUTAP:

Applicable only if restarting from a restart tape

- 0, No new tape written
- 1, Complete ~~new tape~~ created and additional restart ~~data~~ written

INPRT:

0, Write restart tape only at $P = P_{MAX}$ (i.e., at maximum load)

N, Write restart tape every N increments of load

III. Node Specification (I6I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and $-n$ consecutively is the equivalent of the specification of nodes $m, m+1, m+2, \dots, n$ and specifying $m, -p$, and $-n$ consecutively is the equivalent of the specification of nodes $m, m+p, m+2p, \dots, m+kp$ where $m+kp$ is the highest integer of the form less than or equal to n . For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Member Connectivity (Node Numbers of Each Member)
FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type:

- TRIM - Triangular membrane element (Ref. 3)
- BEAM - Beam element
- STRG - Stringer element (Ref. 4)
- TRIP - Triangular plate bending element (with
membrane effects) (Ref. 5)

The first integer field designates the member number. The next integer fields designate the connecting nodes as follows:

TRIM - The nodes for the triangular family of elements are specified around the perimeter beginning with a major (vertex) node, and followed by a minor (midside) node and then alternatively major and minor as shown in Fig. 1. The absence of a minor node must be indicated by a zero or blank field in the proper position.

BEAM - Three node specifications are necessary for the beam element.

Nodes *i* and *j* (Fig. 2) which designate the element end points and a third node *k*, defining the normal to the beam axis about which the section properties are specified. This additional node may be a node of the structural idealization or it may be a "fictitious node" specified just for the purpose of defining the beam section properties. This is shown in Fig. 2.

STRG - Three node specifications are necessary for the stringer element. Nodes *i* and *j*, connecting the end points and if desired an additional node designating a midpoint node. This is shown in Fig. 3. A zero or blank for the midpoint node specifies a two-node stringer. The midside node is the third node specified.

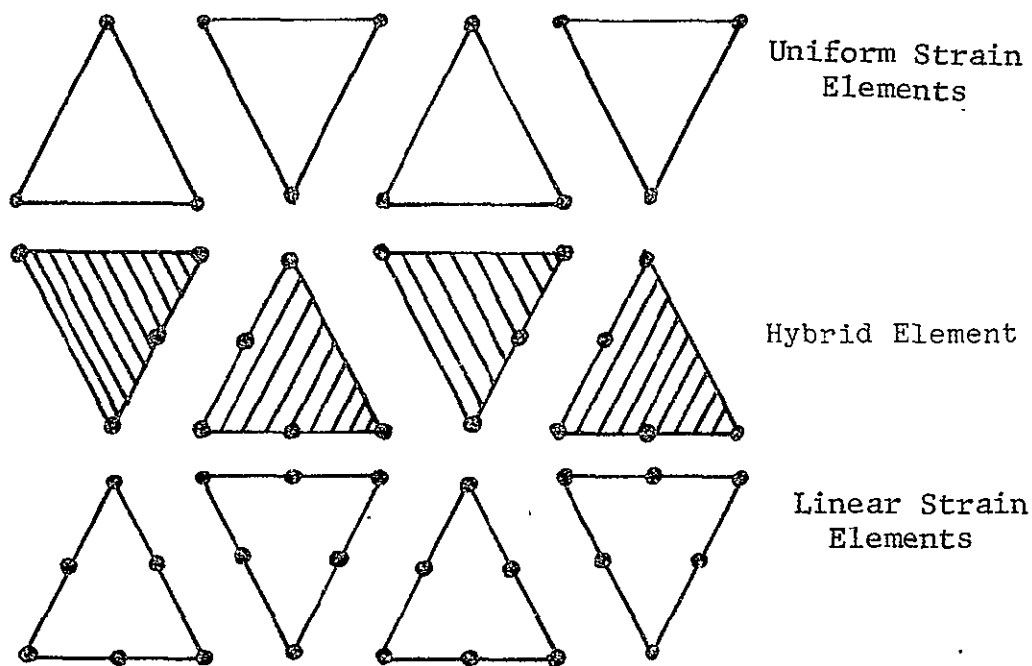


Fig. 1a Triangular Family of Finite Elements Used

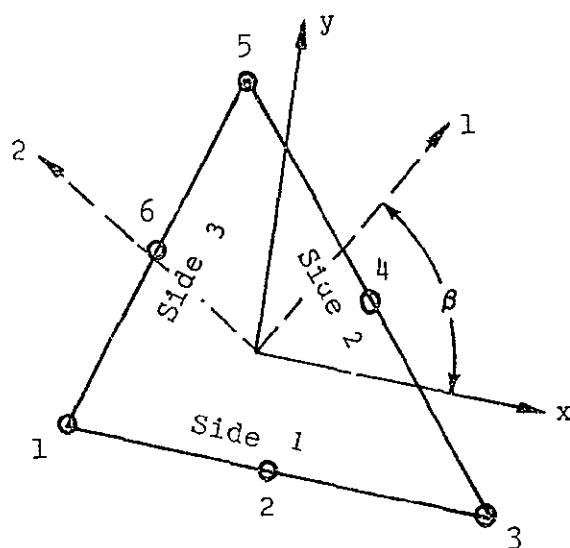


Fig. 1b Elements Topology

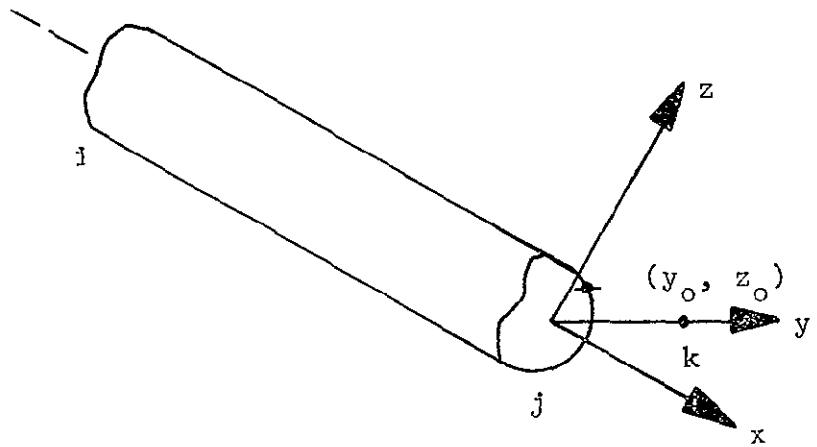


Fig. 2 Beam Element

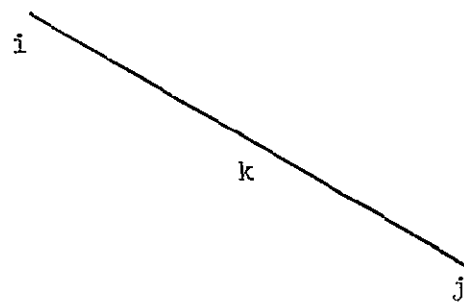


Fig. 3 Stringer Element

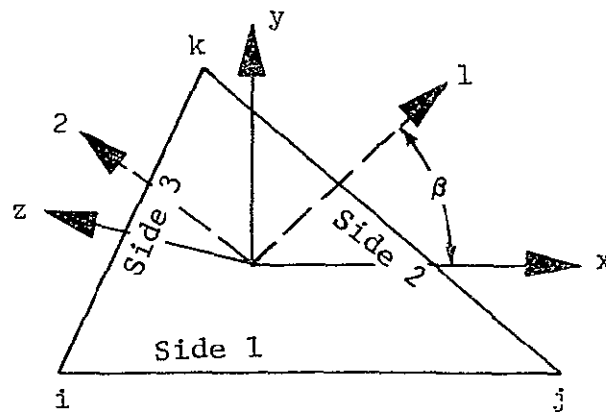


Fig. 4 Triangular Plate Element - Bending and Membrane

TRIP - Three nodes are used to specify the plate element i, j, k as shown in Fig. 4.

If a planar structure is being analyzed using triangular plate and membrane elements, stress and strain output can be calculated with respect to the global coordinate system rather than the element local system by specifying one of the following cards:

GLXY - Structure in the x-y plane

GLXZ - Structure in the x-z plane

GLYZ - Structure in the y-z plane

SEND - Ends the section.

V. X-Coordinates of Nodes FORMAT (E15.7, 13I5)

The x-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given x-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.

VIII. Boundary Conditions FORMAT (12I1, 3X, 13I5, /, 15X, 13I5)

The first twelve fields give the boundary conditions specifications in the order: $u, v, w, \theta_x, \theta_y, \theta_z, \epsilon_x, \epsilon_y, \epsilon_{xy}, \kappa_x, \kappa_y, \kappa_{xy}$; where u, v, w are global displacements in the x, y, z directions, respectively; $\theta_x, \theta_y, \theta_z$ are global rotations with respect to the x, y, z axes; $\epsilon_x, \epsilon_y, \epsilon_{xy}$ are membrane components of the local membrane strain tensor; $\kappa_x, \kappa_y, \kappa_{xy}$ are components of the local curvature tensor. The last six fields for the local membrane strains and curvatures (higher order degrees of freedom) are only used if a plate element is attached to the node. Both tensors are calculated in the local coordinate system (see Fig. 4) of the plate element. This local coordinate system is defined by the first plate element connecting that node that appears in the input stream. For example,

TRIP	3	6	3	2	} Node 3 is not listed previously in the element input
TRIP	4	3	6	7	
TRIP	5	7	4	3	

The tensor quantities at node 3 are all defined in the local coordinate system of element 3. See Appendix IV for examples of plate boundary conditions.

The rotational degrees of freedom $\theta_x, \theta_y, \theta_z$ are applicable only for beam and plate elements. If nonzero values are given to rotational or higher order degrees of freedom at a node and beam and/or plate elements are not attached to the node then a singular stiffness matrix will result.

Zero denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 13I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 13I5 fields are used on a continuation card. A zero or blank I5 field terminates the card scan for a given boundary condition specification. Note: If the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default (i.e., in the absence of beams and plates, 111000000000) with all nodes used in the problem specified. In the absence of plate elements the default is automatically 111111000000.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacements of the form:

$$1) \quad \delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \cdots \alpha_n \delta_{jn}$$

where δ_i is a dependent degree of freedom,
 $\delta_{j1} \cdots \delta_{jn}$ are independent degrees of freedom,
 and $\alpha_1, \alpha_2 \cdots \alpha_n$ are coefficients.

2) Rotation of displacements at a node

$$\delta_i = \alpha_1 \tilde{\delta}_i + \alpha_2 \tilde{\delta}_j + \alpha_3 \tilde{\delta}_k$$

$$\delta_j = \beta_1 \tilde{\delta}_i + \beta_2 \tilde{\delta}_j + \beta_3 \tilde{\delta}_k$$

$$\delta_k = \gamma_1 \tilde{\delta}_i + \gamma_2 \tilde{\delta}_j + \gamma_3 \tilde{\delta}_k$$

where the α , β , γ 's are the direction cosines of the rotation, δ_i , δ_j , δ_k are the displacements with respect to the original global directions and $\tilde{\delta}_i$, $\tilde{\delta}_j$, $\tilde{\delta}_k$ are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

$$\delta_i = \alpha_1$$

where the coefficient α_1 is the applied generalized displacement.

The first two fields designate a node number and a degree of freedom (i.e., 1-12). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material and Section Properties

The format for this input is dependent upon the member type. Each type of input begins with a word of up to four characters.

- MAT1 Material properties for plane stress, necessary with membrane triangles (TRIM) and bending triangles (TRIP). For use with initially isotropic materials which exhibit perfectly plastic or strain hardening behavior and initially orthotropic materials that exhibit perfectly plastic behavior.
- MAT2 Material properties for plane stress, necessary with membrane triangles (TRIM) and bending triangles (TRIP) with initially isotropic or orthotropic behavior that require orthotropic kinematic hardening theory
- MBM Material properties for a beam element
- MSTG Material and section properties for a stringer
- THIK Member thickness for triangular membrane (TRIM) and plate (TRIP) elements
- MBET Angle between local axes and principal directions of orthotropy for TRIM and TRIP elements (see Figs. 1b and 4)
- NLRS Layer and layer output information for TRIP elements
- SREC Beam section properties for a solid rectangular section

SCIR	Beam section properties for a solid circular section
ZSEC	Beam section properties for a Z-section
ISEC	Beam section properties for an I-section
HCIR	Beam section properties for a hollow circular section
HREC	Beam section properties for a hollow rectangular section
LSEC	Beam section properties for an L-section
TSEC	Beam section properties for a T-section
CSEC	Beam section properties for a Channel section
SEND	Ends the section

MAT1 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /,
 2E15.7, /, 4E15.7, /,
 5E15.7, /, (16I5))

The first four cards specify material properties, as follows:

CARD 1: MAT1

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

BETA = No longer used. Set equal to zero.

GONTO = Shear modulus in (1)-(2) principal property plane

VONTO = Poisson's ratio, ν_{12}

$$\text{Note: } \epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2} \sigma_2$$

$$\epsilon_2 = -\frac{\nu_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in
 1-axis direction, α_1

TALF-2. = Coefficient of thermal expansion in
 2-axis direction, α_2

CARD 3: SIGOX = Yield stress in principal 1-direction
 SIGOY = Yield stress in principal 2-direction
 SIGOZ = Yield stress in principal 3-direction
 SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior

If RMOSS = 0; RMOSN = $\bar{\alpha}$ the slope of the linear strain-hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

RMOSS = If RMOSS $\neq$ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$ (see Ref. 7)

Note 1: If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

YLDST = Yield stress in tension

YLDSC = Yield stress in compression

Input for the yield-stress in tension and compression has been maintained in order to accommodate materials that exhibit initial anisotropic plastic behavior. In this case an initial translation of the yield surface is made consistent with the kinematic hardening theory.[†]

Note: Only initially isotropic materials can be treated when considering linear or nonlinear strain hardening using this property card.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

[†]Isakson, G., Armen, H. Jr., and Pifko, A., "Discrete-Element Methods for the Plastic Analysis of Structures," NASA Contractor's Report NASA CR-803, October 1967.

MAT2 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /,
2E15.7, /, 4E15.7, /,
5E15.7, /, E15.7, /,
(16I5))

The first five cards specify material properties, as follows:

CARD 1: MAT2

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

BETA = No longer used. Set equal to zero.

GONTO = Shear modulus in (1)-(2) principal property plane

VONTO = Poisson's ratio, ν_{12}

$$\text{Note: } \epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2} \sigma_2$$

$$\epsilon_2 = -\frac{\nu_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2}$$

$$\gamma_{12} = \frac{1}{G_{12}} \tau_{12}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, α_1

TALF-2 = Coefficient of thermal expansion in 2-axis direction, α_2

CARD 3: SIGOX = Yield stress in principal 1-direction

SIGOY = Yield stress in principal 2-direction

SIGOZ = Yield stress in principal 3-direction

SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: $\bar{RMONX} \equiv$ If $\bar{RMOSX} \neq 0$; $\bar{RMONX} \equiv n$, the shape parameter used in the Ramberg-Osgood representation of the $\sigma_1 - \epsilon_1$ component of the stress-strain behavior.

If $\bar{RMOSX} \equiv 0$; $\bar{RMONX} \equiv \bar{\alpha}_1$ the slope of the linear strain-hardening stress-strain representation in the 1-direction, i.e., $\bar{\alpha}_1 = \frac{E_{T1}}{E_1}$ where E_{T1} is the tangent modulus of the $\sigma_1 - \epsilon_1$ curve.

$\bar{RMOSX}$ If $\bar{RMOSX} \neq 0$; $\bar{RMOSX} \equiv$ Ramberg-Osgood parameter $\sigma_{0.7}$ in principal 1-direction

$\bar{RMONY}$ Same as $\bar{RMONX}$ in principal 2-direction

$\bar{RMOSY}$ Same as $\bar{RMOSX}$ in principal 2-direction

$\bar{RMONXY}$ Same as $\bar{RMONX}$ for shear component in 1-2 plane; $\tau_{12} - \gamma_{12}$ stress-strain curve

CARD 5: $\bar{RMOSXY}$ Same as $\bar{RMOSX}$ for shear component in 1-2 plane; $\tau_{12} - \gamma_{12}$ stress-strain curve

Note: No component of the stress-strain curves may be ideally plastic, i.e., $\bar{RMONX}$ or $\bar{RMONY}$ or $\bar{RMONXY} = 0$ is not allowed. If either is zero all must be zero and a MAT1 card should be used. The Ramberg-Osgood representation of the stress-strain curve is given by

$$\sigma = \frac{\sigma}{\tau} + \frac{3\sigma}{7E} \left(\frac{\sigma}{\sigma_{0.7}} \right)^{n-1}$$

$$\gamma = \frac{\tau}{G} + \frac{3\tau}{7G} \left(\frac{\tau}{\tau_{0.7}} \right)^{n-1}$$

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MBM - Beam Material Properties - FORMAT (A4, 1X, 5E15.7, /, E15.7, /,
(16I5))

CARD 1: MBM

E = Young's modulus

ANU = Poisson's ratio

RMOSS = If not equal to zero, RMOSS equals
Ramberg-Osgood parameter, $\sigma_{0.7}$

RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior. If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the linear strain-hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

YLDST = Yield stress

CARD 2: TALF[—] = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

MSTG - Stringer Properties - FORMAT (A4, 1X, 5E15.7, / E15.7 /, (16I5))

CARD 1: MSTG

E = Young's modulus

A = Cross-sectional area

RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the linear strain-hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

RMOSS = If RMOSS $\neq$ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note 1: If RMOSN = .0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

YLDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

THIK - Element Thicknesses - FORMAT (A4, 1X, E15.7, /, (16I5))
(Necessary with membrane elements
TRIM) and plate elements (TRIP))

CARD 1: THIK

THICK = Element thickness

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

MBET - Orientation of Axes of Material Anisotropy - FORMAT (A4, 1X,
E15.7, /, (16I5))

CARD 1: MBET

BETF = Angle β in degrees between local x-axis
and principal 1-axis for material ortho-
tropy. See Figs. 1b and 4. Only appli-
— cable for TRIM and TRIP elements

Note: This card is an optional card. The
default is BETF = 0.0 for all members.
It should be used with initially ortho-
tropic materials or those exhibiting
orthotropic strain-hardening character-
istics.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

NLRS - Layer and Print Option for TRIP Elements - FORMAT (A4, 1X, /,
2I5)

CARD 1: NLRS

CARD 2: NLRS = Number of layers through the thickness at
which stresses are calculated for TRIP
elements. NLRS must be an even number
 ≤ 20

NPLT = Print out option:

= 0; Print stresses, strains, etc. only at
top and bottom surfaces

= 1; Print stresses, strains, etc. at each
layer through the thickness as well.

Note: This is an optional card. The
defaults are NLRS = 10 and
NPLT = 0. If one is changed
both should be specified. This
card controls option for all
TRIP elements.

Beam Section Properties - FORMAT (A4, 1X, 5E15.7, /, 3E15.7, /,
formats for Cards 3 and 4 (see
below), /, (16I5))

The cards specifying beam section properties all start with
the following information:

CARD 1: SREC }
 SCIR }
 ISEC }
 ZSEC } Defined at the beginning of the section.
 HCIR } The different cross sections are shown
 HREC } in Tables 1 and 2.
 LSEC }
 TSEC }
 CSEC }

A = cross-sectional area
I_{yy} = moment of inertia about y axis (see Fig. 5)
I_{zz} = moment of inertia about z axis (see Fig. 5)
I_{yz} = product of inertia
J = torsional rigidity

If the quantities, A, I_{yy}, I_{zz}, I_{yz} are input as zero or
blank then these quantities are calculated automatically.

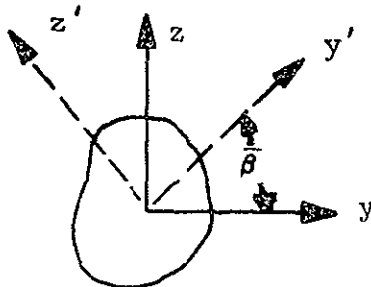


Fig. 5 Definition of Coordinate Axes in
Cross Section of Beam Element

CARD 2: Y_0 = Eccentricity of attachment point in the y_0 direction
 Z_0 = Eccentricity of attachment point in the z_0 direction
 $\bar{\beta}$ = angle defining the transformation of the I_y, I_z, I_{yz} to another reference axis
 (see Fig. 5)

Additional cards are required according to which section is specified. The notation for each section is shown in Table 1.

SREC - FORMAT (2E15.7)

CARD 3: A = Width
 B = Depth

SCIR - FORMAT (E15.7)

CARD 3: R = Radius

ZSEC, ISEC, and CSEC - FORMAT (3E15.7, /, 3E15.7)

CARD 3: A1 = Dimension of upper flange (parallel to y-axis)
 A2 = Dimension of web (parallel to z-axis)
 A3 = Dimension of lower flange (parallel to y-axis)

CARD 4: T1 = Thickness of upper flange
 T2 = Thickness of web
 T3 = Thickness of lower flange

HCIR - FORMAT (2E15.7)

CARD 3: R = Outer radius
 T = Thickness

HREC, LSEC, and TSEC - FORMAT (4E15.7)

CARD 3: A1 = Width (parallel to y-axis)
A2 = Depth (parallel to z-axis)
T1 = Thickness of upper and/or lower flanges
T2 = Thickness of vertical webs

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

SEND Ends the section.

TABLE 1 SECTION INPUT AND GEOMETRY

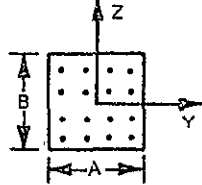
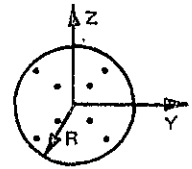
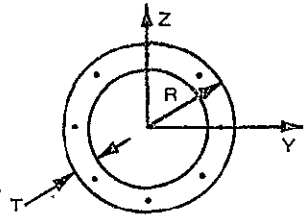
Section	Keyword	Input	
Solid Rectangle	SREC	Area, moments of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o ; transformation angle $\bar{\beta}$. Section width and height, a, b.	
Solid Circular	SCIR	Area, moments of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o ; transformation angle $\bar{\beta}$. Section radius, r.	
Hollow Circular Section	HCIR	Area, moment of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o ; transformation angle $\bar{\beta}$. Outer radius, r, thickness, t.	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

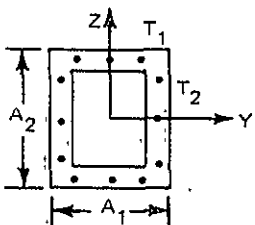
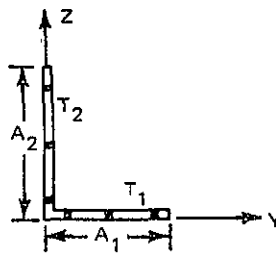
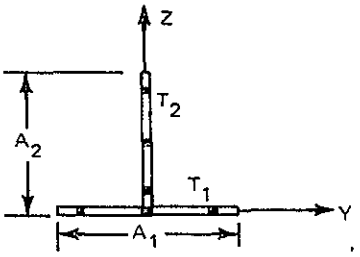
Section	Keyword	Input	
Hollow Rectangular Section	HREC	Area, moment of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Width, a_1 , depth, a_2 , thickness of upper and lower flanges, t_1 , thickness of vertical webs, t_2	
L-Section	LSEC	Area, moment of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Dimension of flange and web a_1, a_2 thickness flange, and web, t_1 , t_2 .	
T-Section	TSEC	Same as for L-Section	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

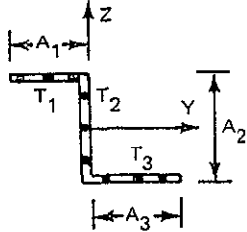
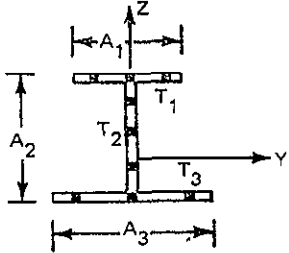
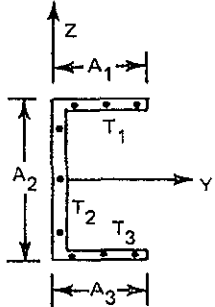
Section	Keyword	Input	
Z-Section	ZSEC	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Dimension of upper and lower flange, a_1, a_3. Dimension of web, a_2, thickness of upper and lower flange, t_1, t_3, thickness of web, t_2 NOTE: $a_1 = a_3$; $t_1 = t_3$	
I-Section	ISEC	Same as Z-section $a_1 \neq a_3$; $t_1 \neq t_3$	
Channel Section	CSEC	Same as Z-Section $a_1 = a_3$; $t_1 = t_3$	

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS

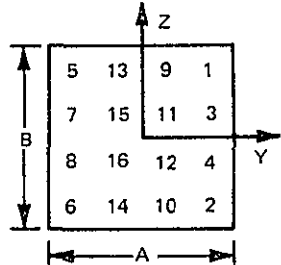
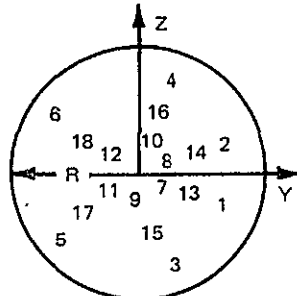
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
Solid Rectangle	SREC	16	$\pm y$	$\pm z$	 Shear Center at Centroid
			1	0.430568a	0.430568b
			3	0.430568a	0.169991b
			9	0.169991a	0.430568b
			11	0.169991a	0.169991b
Solid Circular	SCIR	18	r	$\pm \epsilon^\circ$	 Shear Center at Centroid
			2	0.887a	12.1
			4	0.887a	60.98
			6	0.887a	137.05
			8	0.1127a	12.1
			10	0.1127a	60.98
			12	0.1127a	137.05
			14	0.500a	12.1
			16	0.500a	60.98
			18	0.500a	137.05

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

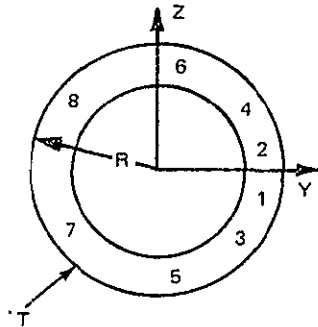
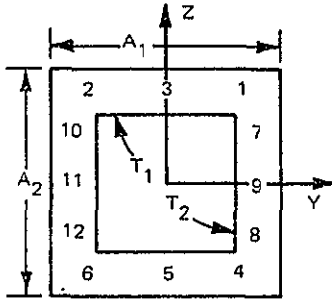
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
Hollow Circular Section	HCIR	8	r	θ°	 Shear Center at Centroid
			1	a	352.8
			2	a	7.2
			3	a	323.4
			4	a	36.6
			5	a	274.6
			6	a	85.4
			7	a	213.0
			8	a	147.0
Hollow Rectangular Section	HREC	12	$\pm y$	$\pm z$	 Shear Center at Centroid
			1	$0.77459\bar{a}_1$	$0.5\bar{a}_2$
			3	0	$0.5\bar{a}_2$
			7	$0.5\bar{a}_1$	$0.77459\bar{a}_2$
			9	$0.5\bar{a}_1$	0
			$\bar{a}_1 = a_1 - t_2$ $\bar{a}_2 = a_2 - t_1$		

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

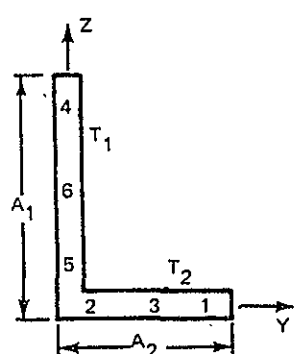
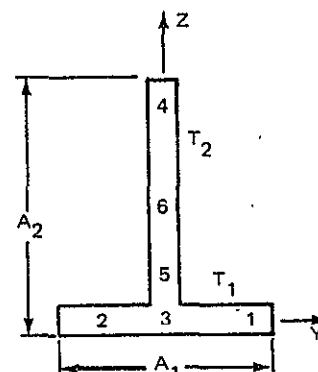
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points	
L-Section	LSEC	6	y	z		
			1	$0.88730\bar{a}_2$		0.0
			2	$0.11271\bar{a}_2$		0.0
			3	$0.50000\bar{a}_2$		0.0
			4	0.0		$0.88730\bar{a}_1$
			5	0.0		$0.11271\bar{a}_1$
			6	0.0		$0.50000\bar{a}_1$
			$\bar{a}_1 = a_1 - \frac{t_2}{2}$, $\bar{a}_2 = a_2 - \frac{t_1}{2}$			
T-Section	TSEC	6	y	z		
			1	$0.77459a_1$		0.0
			2	$-0.77459a_1$		0.0
			3	0.0		0.0
			4	0.0		$0.88730\bar{a}_2 + \frac{t_1}{2}$
			5	0.0		$0.11271\bar{a}_2 + \frac{t_1}{2}$
			6	0.0		$0.50000\bar{a}_2 + \frac{t_1}{2}$
			$\bar{a}_2 = a_2 - t_1$			

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

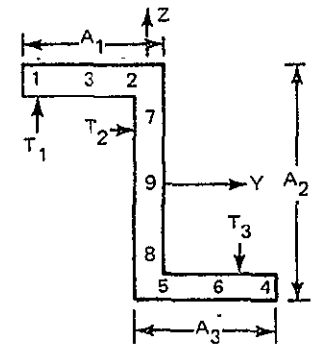
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
Z-Section	ZSEC	9	y	z	 Shear Center at Centroid
			1	$-0.88730a_1$	$\bar{a}_2$
			2	$-0.11271a_1$	$\bar{a}_2$
			3	$-0.5000a_1$	$\bar{a}_2$
			4	$0.88730a_1$	$\bar{a}_2$
			5	$0.11271a_1$	$\bar{a}_2$
			6	$0.5000a_1$	$\bar{a}_2$
			7	0.0	$0.77459a'_2$
			8	0.0	$0.77459a'_2$
			9	0.0	0.0
			$\bar{a}_2 = (a_2 - t_1)/2$, $t_1 = t_3$		
			$a'_2 = (a_2 - t_1)$, $a_1 = a_3$		

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

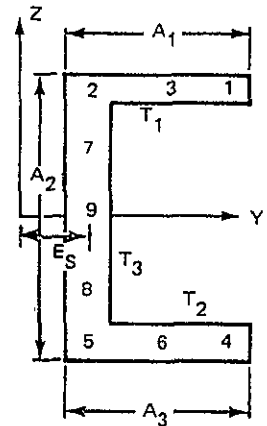
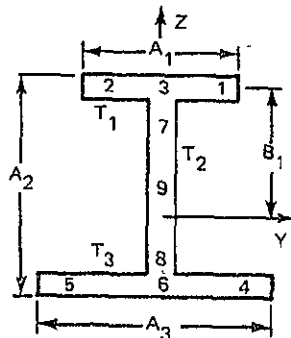
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points	
Channel Section	CSEC	9	y	z	 Axes at Shear Center	
			1	$0.88730\bar{a}_1 + e_s$		$0.5\bar{a}_2$
			2	$0.11271\bar{a}_1 + e_s$		$0.5\bar{a}_2$
			3	$0.5000\bar{a}_1 + e_s$		$0.5\bar{a}_2$
			4	$0.88730\bar{a}_1 + e_s$		$-0.5\bar{a}_2$
			5	$0.11271\bar{a}_1 + e_s$		$-0.5\bar{a}_2$
			6	$0.5000\bar{a}_1 + e_s$		$-0.5\bar{a}_2$
			7	e_s		$0.77459\bar{a}_2$
			8	e_s		$-0.77459\bar{a}_2$
			9	e_s		0.0
			e_s - distance from shear center to web center line			
			$e_s = \frac{1/2(a_1 - t_2)}{1 + 1/6 \left(\frac{t_2(a_2 - t_1)}{t_1(a_1 - t_2)} \right)}, \quad \bar{a}_2 = a_2 - t_1$			
			NOTE: $t_1 = t_3$; $a_1 = a_3$			

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
I-Section	ISEC	9	y	z	 Axes at Shear Center
			1	$0.774569a_1$	b_1
			2	$-0.77459a_1$	b_1
			3	0.0	b_1
			4	$0.77459a_3$	b_2
			5	$0.77459a_3$	b_2
			6	0.0	b_2
			7	0.0	$0.8873\bar{b}_1 + 0.11271\bar{b}_2$
			8	0.0	$0.11271\bar{b}_1 + 0.8873\bar{b}_2$
			9	0.0	$0.5\bar{b}_1 + 0.5\bar{b}_2$
			$b_1 = \frac{-\frac{t_3}{a_2} \left(\frac{a_3}{t_1} \right) \left(\frac{a_3}{a_1} \right)^3}{\left[1 + \left(\frac{t_3}{t_1} \right) \left(\frac{a_3}{a_1} \right) \right]^3}, \quad b_2 = b_1 \left(\frac{t_1}{t_3} \right) \left(\frac{a_1}{a_3} \right)^3$		
			$\bar{a}_2 = a_2 - 0.5(t_1 + t_3) ; \quad \bar{b}_1 = b_1 - 0.5t_1 ; \quad \bar{b}_2 = b_2 - 0.5t_3$		

XI. Applied Loads

Many different types of loading are currently admissible — concentrated loads at nodes, distributed loads on an edge of a triangular member, distributed surface loads, distributed line loads on beams, and thermal loads. Each loading situation is designated by one of the following four-character key words:

CONC	(Concentrated force or moment at a node)
TRIA	(Distributed edge load, TRIM elements)
SURF	(Distributed surface loads, TRIP element)
EDG1	(Distributed edge load, side 1, TRIP element)
EDG2	(Distributed edge load, side 2, TRIP element)
EDG3	(Distributed edge load, side 3, TRIP element)
BMLO	(Distributed line load, beam element)
TMPU	(Temperatures at upper surface for TRIP element or temperatures for TRIM element)
TMPL	(Temperatures at middle surface for TRIP element)
TMPL	(Temperatures at lower surface for TRIP element)
SEND	(Section end)

Note: The keywords for the section on applied loads are used as section headings. The keyword appearing on an input card designates that the input to follow is associated with a particular type of applied loading. A blank card (as described in succeeding sections) delimits the input and indicates that the next card contains a different keyword. For example, CONC indicates input that follows is for concentrated loads and TRIA; distributed loads on an edge of a membrane triangle. Thus the input is as follows:

CONC

data
for
concentrated
loads

blank card

TRIA

data
for
distributed
loads

blank card

SEND

It should be noted that the keywords can appear in any order and may be specified more than one time in the applied load section.

TRIA - Distributed edge load (TRIM elements) - FORMAT (A4, /, 4I5, 3E15.7, /, 15X, 3E15.7)

Each card gives the load components applied at a member side (in units of force) as follows:

CARD 1: TRIA

CARD 2: First I5 field:	Number of node (m)
Second I5 field:	Number of minor node on the side, zero, or blank if there is no minor node

Third I5 field:	Number of other end point node (n)
Fourth I5 field:	NODR - Reference node number
First E15.7 field:	x load component at node m
Second E15.7 field:	y load component at node m
Third E15.7	z load component at node m
CARD 3: First E15.7 field:	x load component at node n
Second E15.7 field:	y load component at node n
Third E15.7 field:	z load component at node n

Note: If NODR = 0 then x, y, z are global directions. If NODR $\neq$ 0 then the x-direction is parallel to the element edge specified by m - n; the y-direction is perpendicular to x in the plane defined by the three points, m, n, NODR; the z-axis is perpendicular to the x-y plane.

The program allows for a linearly varying distributed load from node to node. As many load components as desired may be specified. A blank card (i.e., zero or blank first I5 field) ends the specification of distributed edge loads for TRIM elements.

CONC - Concentrated loads - FORMAT - (A4, /, I5, 3E15.7, /, 5X,
3E15.7)

Each card gives the load components in global directions at a specified node.

CARD 1: CONC

CARD 2: I5 field: . Node number

First E15.7 field: Force component F_x

Second E15.7 field: Force component F_y

Third E15.7 field: Force component F_z

CARD 3: First E15.7 field Moment component M_x

Second E15.7 field Moment component M_y

Third E15.7 field Moment component M_z

A blank card (i.e., zero or blank first I5 field) ends the specification of concentrated loads.

SURF - Distributed Surface Loads (TRIP element) - FORMAT (A4, 1X,
3E15.7, /, 3E15.7, /,
3E15.7, /, (16I5))

CARD 1: SURF

PXI Value of distributed surface load in force/
unit area in local element x-direction at
node i of element (see Fig. 4).

PYI Value of distributed surface load in force/
unit surface area in local y-direction at
node i of element

PZI Value of distributed surface load in force/
unit surface area in local z-direction at
node i of element

CARD 2: PXJ Same as PXI but at node j
PYJ Same as PYI but at node j
PZJ Same as PZI but at node j

CARD 3: P XK Same as PXI but at node k
PYK Same as PYI but at node k
PZK Same as PZI but at node k

Note: A linear variation of the surface loads
in the plane of the element is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

BMLO - Distributed Line Load, Beam Element - FORMAT (A4, /, 4E15.7, /,
(16I5))

Card 1: BMLO

CARD 2: PYI Force/unit length in local y-direction (see
Fig. 5) at node i
PYJ Force/unit length in local y-direction at
node j
PZI Force/unit length in local z-direction at
node i
PZJ Force/unit length in local z-direction at
node j

Note: A linear variation of the distributed
load between nodes is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing. A blank card ends this section.

EDG1 - Distributed Edge Load, Side 1, TRIP Element - FORMAT (A4, 1X,
3E15.7, /, 3E15.7,
/, (16I5))

This card inputs the distributed edge or line loads along side 1 of a TRIP element. Side 1 is the side connecting nodes i and j (see Fig. 4).

CARD 1: EDG1

PXI Local x-component of force/unit length at
ith node

PYI Local y-component of force/unit length at
ith node

PZI Local z-component of force/unit length at
ith node

CARD 2: PXJ Same as PXI but at node j

PYJ Same as PYI but at node j

PZJ Same as PZI but at node j

Note: A linear variation of the edge loads
is assumed between nodes i and j.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

EDG2 - Distributed Edge Load, Side 2, TRIP Element - FORMAT (A4, 1X,
3E15.7, /, 3E15.7
/, (16I5)) .

This section inputs the distributed edge or line loads along side 2 of a TRIP element. Side 2 is the side connecting nodes j and k (see Fig. 4). All input is the same as in the EDG1 section with EDG2 replacing EDG1 as the alphameric clue word and "j" replacing "i" and "k" replacing "j" in the description.

EDG3 - Distributed Edge Load, Side 3, TRIP Element - FORMAT (A4, 1X,
3E15.7, /, 3E15.7,
/, (16I5))

This section inputs the distributed edge or line loads along side 3 of a TRIP element. Side 3 is the side connecting nodes k and i (see Fig. 4). All input is the same as in the EDG1 section with EDG3 replacing EDG1 as the alphameric clue word and "k" replacing "i" and "i" replacing "k" in the description.

TMPU - Nodal Temperatures At Upper Surface for TRIP Element or
Nodal Temperatures for TRIM Element - FORMAT (A4, /, E15.7,
13I5, /, (15X, 13I5))

This section is used to input temperatures at nodes of TRIM elements and at the upper surface (i.e., $z = +h/2$, where h is the plate thickness) of TRIP plate elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the lower surface and middle surface use the TMPL and TMPM cards, respectively.

CARD 1: TMPU

CARD 2: E15.7 field Temperature at node (TRIM)
or temperature at upper surface
of node (TRIP)

I5 Integer fields Applicable nodes

The temperatures of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given temperature. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the TMPU input. Both shorthand representations of Section III are allowed.

Note: For the TRIM element the nodal temperatures are averaged to obtain one element temperature. For the TRIP element a linear in-plane variation of the temperatures from node to node is assumed.

TMPM - Nodal Temperatures at Middle Surface for TRIP Element -

FORMAT (A4, /, E15.7, 13I5, /,
(15X, 13I5))

This section is used to input temperatures at nodes at the middle surface ($z = 0$) of TRIP plate elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the upper and lower surface use the TMPU and TMPL cards, respectively.

The input formats and information for the card are identical to the TMPU card with TMPM replacing TMPU.

TMPL - Nodal Temperatures at Lower Surface for TRIP Element -

FORMAT (A4, /, E15.7, 13I5, /,
(15X, 13I5))

This section is used to input temperatures at nodes at the lower surface ($z = -h/2$, h is the plate thickness) of TRIP elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the upper and middle surface use the TMPU and TMPM cards, respectively.

The input formats and information for this card are identical to the TMPU card with TMPL replacing TMPU.

A card with SEND in the first four columns ends the section for applied loading.

XII. Members to be Printed - FORMAT (16I5)

Specifies the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A blank card or card with only zero entries ends the section.

XIII. Nodes to Be Printed - FORMAT (16I5)

Nodes whose displacements are to be printed as per Section XII.

XIV. Parameters for Plastic Analysis - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as a decimal multiple of yield load.

XV. Parameters for Succeeding Load Cycles - FORMAT (I5, 2E15.7)

NPRNT = If equal to zero, no additional load cycle and end of problem. If greater than zero, print output every NPRNT increments.

PMAX = Maximum load for this load cycle

PPCT = Load increment as percentage of yield load.

Note: PPCT should be equal to one-half the original value since the plastic range is twice the elastic range for initial loading.

XVI. Change Plastic Material Properties for New Load Cycle

The input for this section begins with a four-character word as follows:

MAT1 New material properties for plane stress membrane triangles (TRIM) or plate (TRIP) elements (see Section X for applicability)

MAT2 New material properties for orthotropic strain-hardening materials (see Section X for applicability)

MBM New material properties for beam elements

MSTG New material properties for a stringer element

MAT1 - FORMAT (A4, 1X, I5, /, 4E15.7, /, 5E15.7)

CARD 1: MAT1

IMEM One member number from the group that is
to be changed

CARD 2: SIGOX }
SIGOY } Defined in Section X
SIGOZ }
SIGXY }

CARD 3: RMOSN }
RMOSS } Defined in Section X
RMOSE }
YLDST }
YLDSC }

One set of three cards is required for each MAT1 group to be changed. A MAT1 group may be changed to a MAT2 group after a half load cycle.

MAT2 - FORMAT (A4, 1X, I5, /, 4E15.7, /, 5E15.7, /, E15.7)

CARD 1: MAT2

IMEM One member number from the group that is to
be changed

CARD 2: SIGOX }
SIGOY } Defined in Section X
SIGOZ }
SIGXY }

CARD 3: RMONX }
RMOSX } Defined in Section X
RMONY }
RMOSY }
RMONXY }

CARD 4: RMOSXY Defined in Section X

One set of four cards is required for each MAT2 group to be changed. A MAT2 group may be changed to a MAT1 group after a half load cycle.

MBM or MSTG - FORMAT (A4, 1X, I5, /, 3E15.7)

CARD 1: MBM

or

MSTG

IMEM (One member number from the group that is
to be changed)

CARD 2: RMOSN (Ramberg-Osgood parameters defined in Section X).

RMOSS

YLDST

The section is ended with a section end card, SEND.

Note: If there is no change in material properties, a SEND card must appear in the appropriate place in the input stream.

XVII. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVIII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (20A4)

As in Section I

CARD 2: NPNTC, NPRNT, IRESRT, NUTAP, INPRT - FORMAT (5I5)

As in Section II. Here NPNTC is ignored.

If IRESRT is equal to 2

Restart from an elastic critical load using a previously created restart tape

CARD 3: PMAX, PPCT - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

Input is then continued from Section XV

If IRESRT is equal to 3

Continuation from some previously plastic load. The job can be restarted from a previous maximum load or from any intermediate load level.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form;

* * * * *

PLASTIC ANALYSIS VALUES FOR RESTART TAPE HAVE BEEN
WRITTEN AT P = 1.5000000E+03 FOR PRINT INTERVAL 30

* * * * *

In this case NRSRT is equal to 30 and the load is
not necessarily the last maximum value.

Note: For cases when the job ends abnormally (for example when
time or lines are exceeded) the job can be restarted with
a value for NRSRT obtained from the output. In this case
the last value of NRSRT should not be used. This will
insure that a complete set of data will be accessed from
the restart tape.

If IRESRT is equal to 4

Restart from a previous maximum load level
followed by a reversal of a previous load
distribution (CYCLE). A reversal of a
previous load distribution can be placed
on the structure from any previously gener-
ated and saved state of stress.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can
be ignored. If starting from some intermediate load
level, NRSRT is a unique number obtained as output
from the job that generated the restart tape. This
output has the form shown in the preceding dis-
cussion for IRESRT equal to three. The value of
PMAX is ignored.

Succeeding cards follow as in Sections XV through XVII.

XIX. Alternate Input Method

All or part of the input may be read from a tape. This tape is mounted as Unit 23 with the appropriate job control cards. The tape is accessed by specifying the keyword, TAPE (FORMAT A4), in the proper place in the card input stream. Input will then be read from the tape until a SEND (A4) is encountered on the tape. This returns the input reader to the card input stream.

Note: All SEND formats used to end sections must be in card form as the SEND encountered on the tape only sends the reader back to the card input stream. Also blank cards which end sections are best not put on the tape but rather on cards as this allows the user to add card input to tape input, i.e., if a blank ends a section and is encountered on the tape format then the section ends and no new data may be added, but if the blank does not occur on tape the section is still open and can be closed with a blank card in the card input stream.

Example:

Input for member connectivity, Section IV, and coordinates have been generated on a tape in the prescribed manner.

Input cards are as follows:

TITLE

0 5

1 - 100

TAPE (reads data from a tape)

TRIM

· (additional elements if desired)
·
·

SEND

TAPE (reads X-coordinates from tape)
 (Blank card to end section for X-coordinates)

TAPE (reads Y-coordinates from tape)
 (Blank card to end section for Y-coordinates)

5.0 (card input for Z-coordinates)

·
·
·

(Blank card ends section for Z-coordinates)

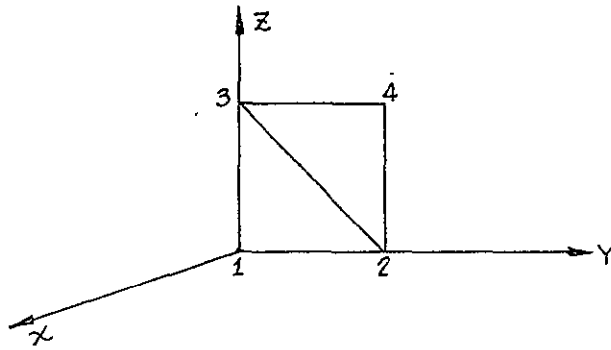
If no additional tape input, remaining card input as usual.

REFERENCES

1. Prager, W., "The Theory of Plasticity: A Survey of Recent Achievements," (James Clayton Lecture) Proc. Inst. Mech. Engrs., Vol. 169, 1955, p. 31.
2. Ziegler, H., "A Modification of Prager's Hardening Rule," Quart. Appl. Math., Vol. 17, No. 1, 1959, p. 55.
3. Armen, H., Jr., Pifko, A., and Levine, H., "Finite Element Analysis of Structures in the Plastic Range," NASA Contractor Report CR-1649, February 1971.
4. De Veubeke, B. F., "Displacement and Equilibrium Models in the Finite Element Method," in Stress Analysis, edited by Zienkiewicz and Hollister, John Wiley and Sons, Ltd., 1965, p. 145.
5. Armen, H., Jr., Levine, H., Pifko, A., and Levy, A., "Nonlinear Analysis of Structures," NASA Contractor Report CR-2351, March 1974; also Grumman Research Department Report RE-454, May 1973.
6. Abramowitz, M. and Stegun, I. A., Eds., Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series, 1964.
7. Pifko, A., Levine, H. S., and Armen, H. Jr., "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractors Report NAS CR-2568, November 1975.

APPENDIX I -- MULTIPOINT CONSTRAINTS

EXAMPLE 1



Node 1 and 2 are fixed.

There is an applied displacement at node 3 in the negative y direction. The distance between nodes 3 and 4 remains constant.

Boundary Conditions

011000000000 1-4 default condition

000000000000 1 2 4

021000000000 3

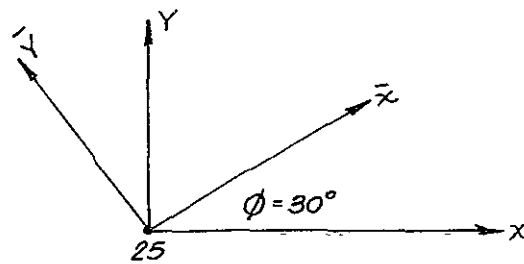
Blank card

Dependencies

Node	Dof	Node	Dof		
3	2	3	2	-.050	(specified displacement)
4	2	3	2	1.0	restraint condition that
4	3	3	3	1.0	3-4 remain rigid

Note: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).

EXAMPLE 2 Rotation of Displacements at a Boundary



For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are x, y . Thus:

$$u = \bar{u} \cos \phi - \bar{v} \sin \phi$$

$$v = \bar{u} \sin \phi + \bar{v} \cos \phi$$

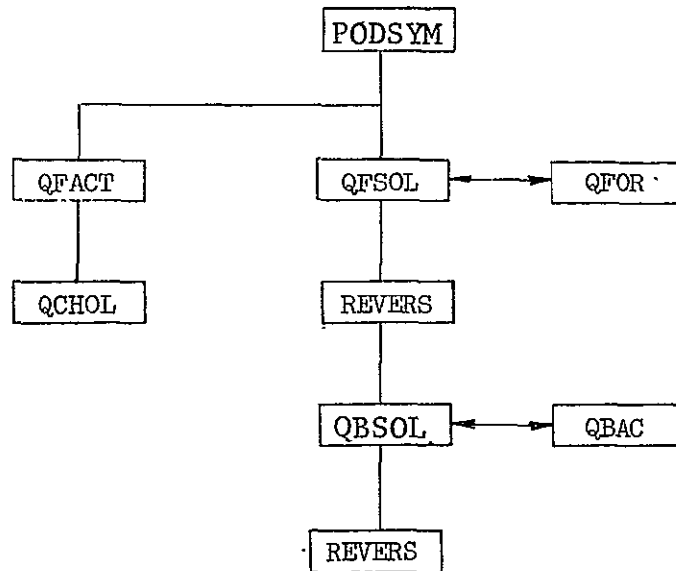
The multipoint constraint cards are:

25	1	25	1	.86602540E00	25	2	-.50E00
25	2	25	1	.50E00	25	2	.8660254E00

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.

APPENDIX II — SOLUTION PACKAGE

PODSYM - Solution of Symmetric Positive Definite Banded Matrix Equations



Problem

Solve $AX = Y$, where

$$A = \begin{bmatrix} \diagup \\ \diagup \\ \diagup \\ \diagup \\ \diagup \end{bmatrix}$$

is a banded positive definite symmetric matrix; X is the desired solution vector; and Y is the known right side (load vector). PODSYM is the user interface and supervisory routine. It uses the Cholesky algorithm which factors the total stiffness matrix into LL^T (where L is a lower triangular matrix) and then solves a pair of triangular sets of equations.

The factorization is carried out by subroutine QFACT which supervised the storage and data set allocation and subroutine QCHOL which generates the lower triangular L matrix. QCHOL implements the Cholesky algorithm to factor the positive definite symmetric A matrix as the product of a lower triangular matrix with its transpose:

$$A = LL^T$$

A straightforward argument establishes the possibility of decomposing any positive definite matrix in this fashion. Once L has been obtained, it is not difficult to solve the system of linear equations $AX = Y$ by calculating Z as the solution of the lower triangular system $LZ = Y$, and then X as the solution of the upper triangular system $L^T X = Z$. The forward solution ($LZ = Y$) is accomplished by subroutines QFSOL and QFOR and the back solution ($L^T X = Z$) by subroutines QBSOL and QBAC. Before the call to QBSOL, subroutine REVERS is called, which reverses the rows of L so that the last row becomes the first row, etc. This is accomplished in order to sequentially access L^T during the back solution.

The above algorithm is noteworthy for its assured stability and general efficiency. It is possible to carry out an error analysis of the procedure as it is represented on a digital computer; such analysis shows that the computed L matrix satisfies an equation of the form

$$A + E = LL^T$$

with bounds on the elements of E which show that E is small compared to A. The effect of rounding errors made in the subsequent solution of $LZ = Y$ and $L^T X = Z$ may then be taken into

account by (implicitly) introducing an additional perturbation F into A , and it is then concluded that the computed solution X_0 exactly satisfies the equation

$$(A = E + F)X_0 = Y$$

Since $E + F$ can be shown to be small, one would like to infer that X_0 is almost an exact solution of the original equations, and unless A is too nearly singular, such a conclusion is indeed warranted. But if A is very ill-conditioned, no such result can be guaranteed, and X_0 may be far from the mathematically correct solution; in this event single-precision computation will not suffice for the calculation of an accurate solution, and since the solution will be very sensitive to small errors in A , it is unlikely that even a high-precision computation will yield satisfactory results unless A and Y are known (and supplied) to more than single-precision accuracy. The PODSYM subroutines make a fairly realistic attempt to detect and report pathological conditions of this sort.

The large positive definite matrices that occur in practical work very often contain a large number of zero entries and the program seeks to benefit from the presence of these elements by modifying the standard Cholesky formulae.

$$l_{kk} = \left[A_{kk} - \sum_{j=1}^{k-1} l_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$l_{ik} = \left[A_{ik} - \sum_{j=1}^{k-1} l_{ij} l_{kj} \right] / l_{kk} \quad \text{for } i > k$$

to read instead

$$\ell_{kk} = \left[A_{kk} - \sum_{j=v(k)}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$\ell_{ik} = \left[A_{ik} - \sum_{j=\mu(i,k)}^{k-1} \ell_{ij} \ell_{kj} \right] / \ell_{kk}$$

APPENDIX III -- CHANGING THE NUMBER OF NODES AND MEMBERS IN BEND

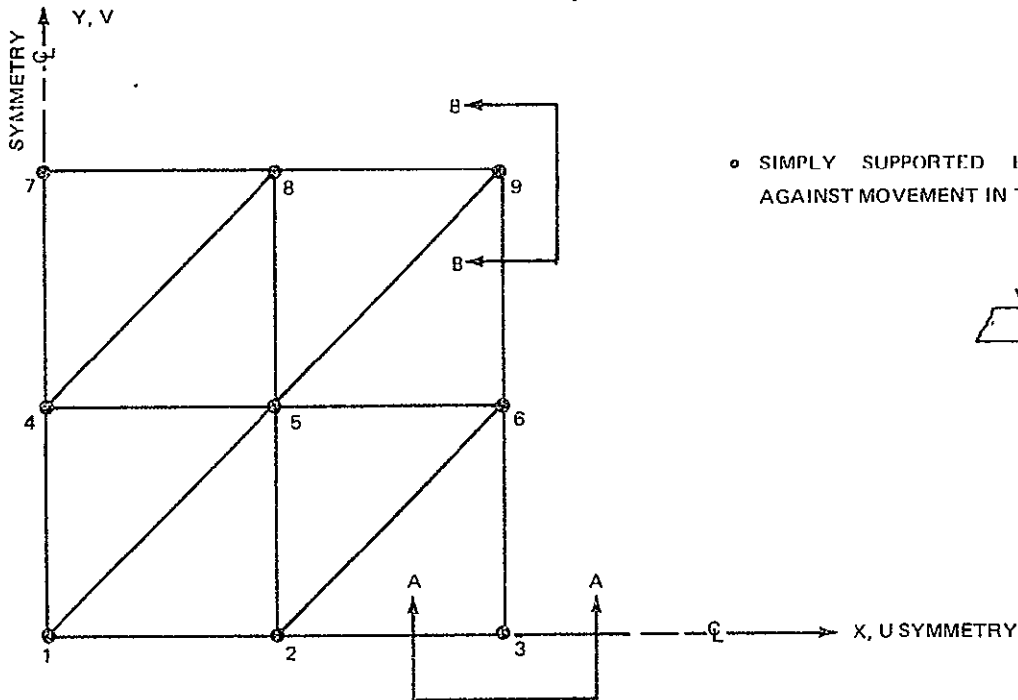
The number of nodes and members allowed for a BEND problem is variable. The following changes must be made in the MAIN program for a successful up-dimensioning or down-dimensioning of nodes and/or members.

- Reset MXMEM equal to the maximum number of members desired
- Reset MXNOD to the maximum number of nodes desired
- Redimension C the main working area so that it is greater than the greater of $10.5 * MXNOD + 6.5 * MXMEM$ or $24 * MXNOD + 1975$
- Set NCORE equal to the dimension of the C array
- For maximum efficiency set MHCON equal to a prime number approximately 1/3 more than MXMEM
- For maximum efficiency set NHCON equal to a prime number approximately 1/3 more than MXNOD. A table of prime numbers appears in Ref. 6, page 870.
- For CDC versions, dimension IRAC (random access array) to MXMEM+1.

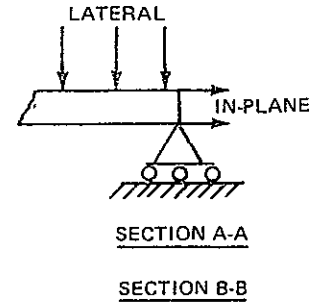
The following limitations on the number of property specifications exist in BEND and can only be changed through redimensioning within the program:

<u>Property or Section</u>	<u>Current Limitation</u>
Number of MAT1 <u>and</u> MAT2 property types	20
Number of MBM property types	20
Number of beam section property types	100
Number of types of edge loads for EDG1, EDG2, EDG3 arrays	100
Number of types of surface loads applied (SURF)	100
Number of multipoint constraint conditions	500
Number of beam element distributed load specification types	100
Number of boundary condition type specifications	100

APPENDIX IV — EXAMPLES OF PLATE BOUNDARY CONDITIONS



- SIMPLY SUPPORTED EDGES NOT RESTRAINED AGAINST MOVEMENT IN THE X-Y PLANE.



BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

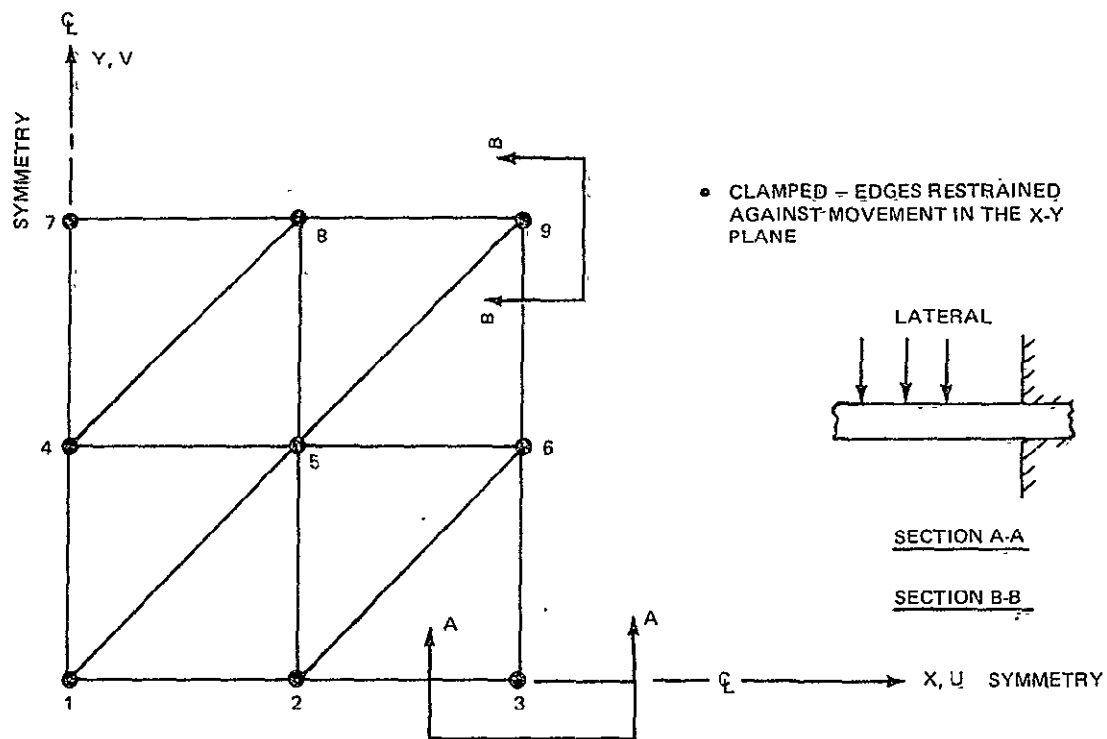
NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	κ_{xx}	κ_{yy}	κ_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	1	0	1	0	1	0	1	1	0	1	1	0
3	1	0	0	0	1	0	1	1	0	0	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	1	1	0	0	1	$0^{(1)}$	1	1	$0^{(2)}$	0	0	1
6	1	1	0	0	1	1	1	1	1	0	0	1
7	0	1	0	1	0	0	1	1	0	0	0	0
8	1	1	0	1	0	$0^{(1)}$	1	1	$0^{(2)}$	0	0	1
8	1	1	0	1	0	1	1	1	1	0	0	1
9	1	1	0	0	0	$0^{(1)}$	1	1	$0^{(2)}$	0	0	1
9	1	1	0	0	0	1	1	1	1	0	0	1

ALL ZERO
IF NO IN-PLANE LOAD

ALL ZERO IF
NO IN-PLANE LOAD

NOTES:

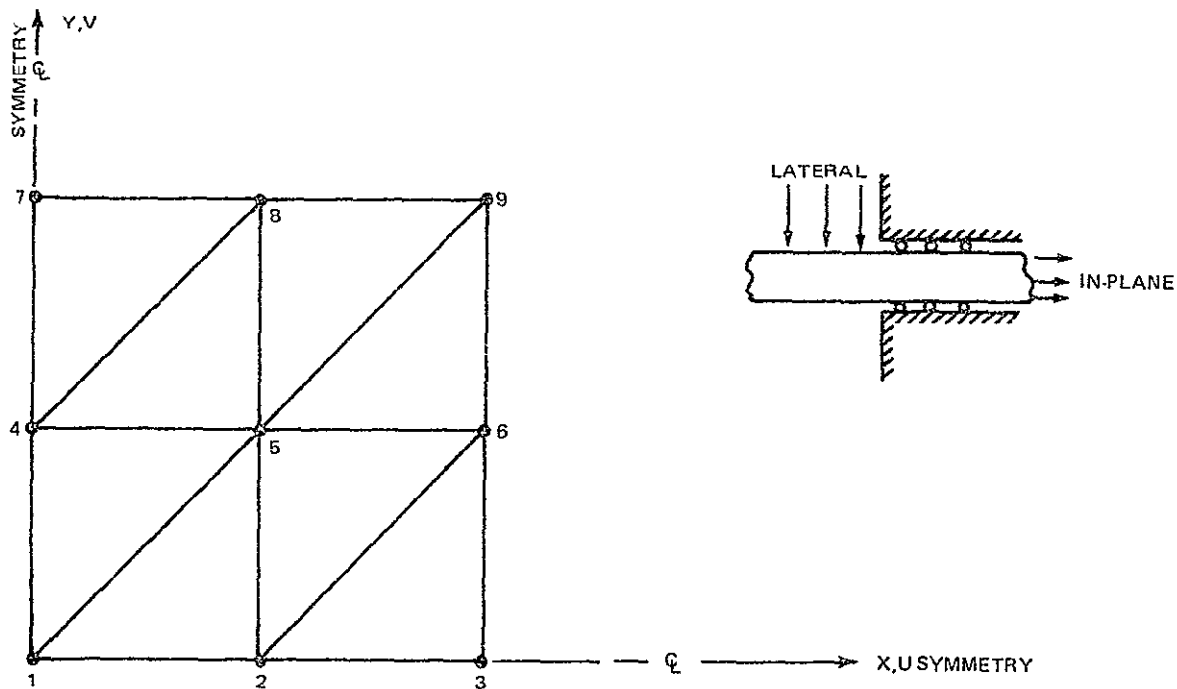
- (1) = 0 FOR ZERO OR UNIFORM DIRECT IN-PLANE LOADS
= 1 FOR NON-UNIFORM IN-PLANE LOADS
 - (2) = 0 FOR ZERO IN-PLANE SHEAR LOAD
= 1 FOR IN-PLANE SHEAR
- NODE 5 COMPLETELY FREE



BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	k_{xx}	k_{yy}	k_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	1	0	1	0	1	0	1	1	0	1	1	0
3	0	0	0	0	0	0	1	0	0	1	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	0	0	0	0	0	0	1	0	1	1	0	0
7	0	0	0	0	0	0	0	1	0	0	1	0
8	0	0	0	0	0	0	0	1	1	0	1	0
9	0	0	0	0	0	0	0	0	0	0	0	0

NODE 5 COMPLETELY FREE



BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	κ_{xx}	κ_{yy}	κ_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	1	0	1	0	1	0	1	1	0	1	1	0
3	1	0	0	0	0	0	1	1	0	1	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	1	1	0	0	0	0 ⁽¹⁾	1	1	0 ⁽²⁾	1	0	0
6	1	1	0	0	0	1	1	1	1	1	0	0
7	0	1	0	0	0	0	1	1	0	0	1	0
8	1	1	0	0	0	0 ⁽¹⁾	1	1	0 ⁽²⁾	0	1	0
8	1	1	0	0	0	1	1	1	1	0	1	1
9	1	1	0	0	0	0 ⁽¹⁾	1	1	0 ⁽²⁾	0	0	0
9	1	1	0	0	0	1	1	1	1	0	0	0

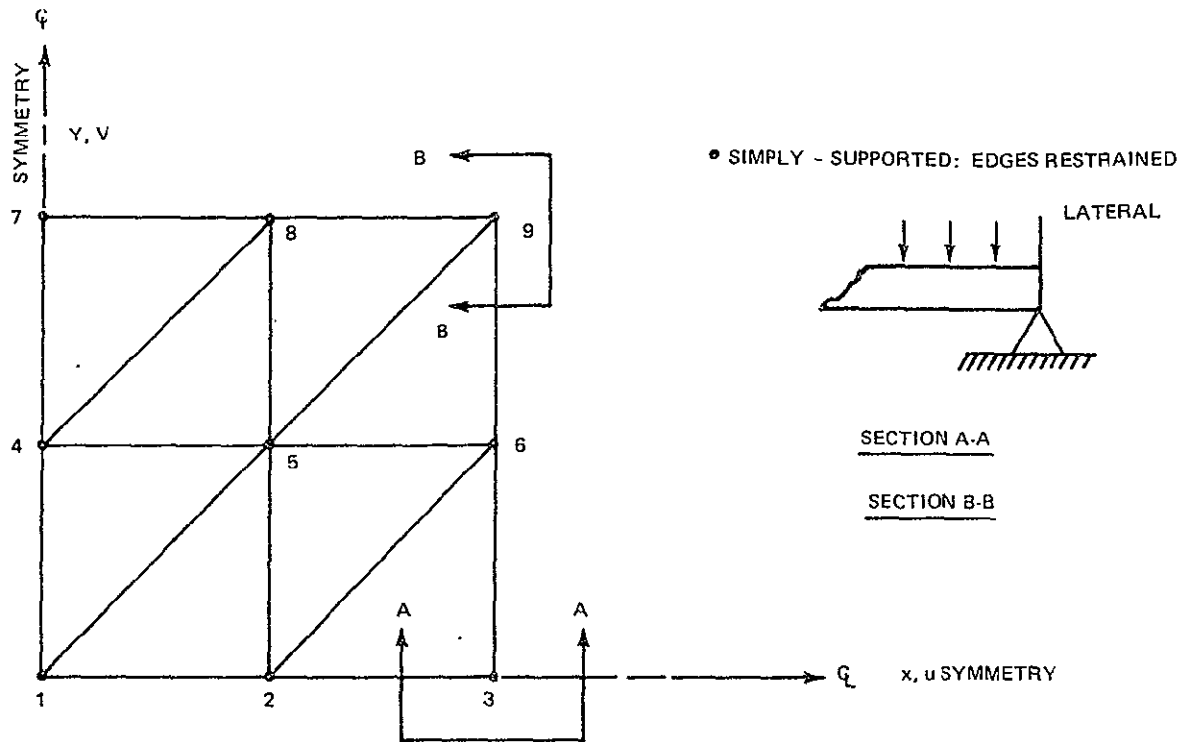
ALL ZERO IF
NO INPLANE LOAD

ALL ZERO IF
NO INPLANE LOAD

NOTES

- (1) = 0 FOR ZERO OR UNIFORM DIRECT IN-PLANE LOADS
= 1 FOR NON-UNIFORM IN-PLANE LOADS
- (2) = 0 FOR ZERO IN-PLANE SHEAR LOAD
= 1 FOR IN-PLANE SHEAR

NODE 5 COMPLETELY FREE



BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	k_{xx}	k_{yy}	k_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	0	0	1	0	1	0	1	1	0	1	1	0
3	0	0	0	0	1	0	1	$1^{(1)}$	0	0	0	0
3	0	0	0	0	1	0	1	$0^{(2)}$	0	0	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	1	$1^{(1)}$	0	0	1	0	1	$1^{(1)}$	$0^{(1)}$	0	0	1
6	1	$0^{(2)}$	0	0	1	0	1	$0^{(2)}$	$1^{(2)}$	0	0	1
7	0	0	0	1	0	0	$1^{(1)}$	1	0	0	0	0
7	0	0	0	1	0	0	$0^{(2)}$	1	0	0	0	0
8	$1^{(1)}$	0	0	1	0	0	$1^{(1)}$	1	$0^{(1)}$	0	0	1
8	$0^{(2)}$	0	0	1	0	0	$0^{(2)}$	1	$1^{(2)}$	0	0	1
9	0	0	0	0	0	0	$1^{(1)}$	$1^{(1)}$	0	0	0	1
9	0	0	0	0	0	0	$0^{(2)}$	$0^{(2)}$	0	0	0	1

NOTES-

(1) WHEN DISPLACEMENTS ARE NOT RESTRAINED IN THE DIRECTION OF THE EDGE

(2) WHEN DISPLACEMENTS ARE FULLY RESTRAINED

NODE 5 COMPLETELY FREE

INSTRUCTIONS FOR USE OF HEX

A Program for the Elastic-Plastic Analysis
of Three Dimensional Solids

INSTRUCTIONS FOR THE USE OF HEX

HEX is a finite element program to treat the elastic, elastic-plastic or elastic-cyclic plastic response of arbitrary three dimensional solid structures. The program uses a family of isoparametric hexahedra elements (Refs. 1 and 2) consisting of eight-node hexahedra and hexahedra with up to 12 additional midside nodes as shown in Fig. 1. Sample problems from the present program can be found in Ref. 3.

The program is capable of treating the elastic and the elastic-ideally plastic response of orthotropic materials. In addition, consideration is given to isotropic materials exhibiting elastic-ideally plastic, linear strain hardening, or nonlinear strain hardening behavior. Further, the kinematic hardening theory of plasticity is used (Refs. 4-6) so that provision for cyclic loading conditions involving reversed plastic deformation is included.

The input to the program is categorized in the following sections:

I. Problem Title FORMAT (20A4)

Any 80-character title describing the problem.

II. NPNTC, NPRNT, IRESRT, NUTAP, INPRT FORMAT (5I5)

$0 \leq \text{NPNTC} \leq 63$:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0 No intermediate printout
 = 1 Print the load vector
 = 2 Print element stiffness matrix

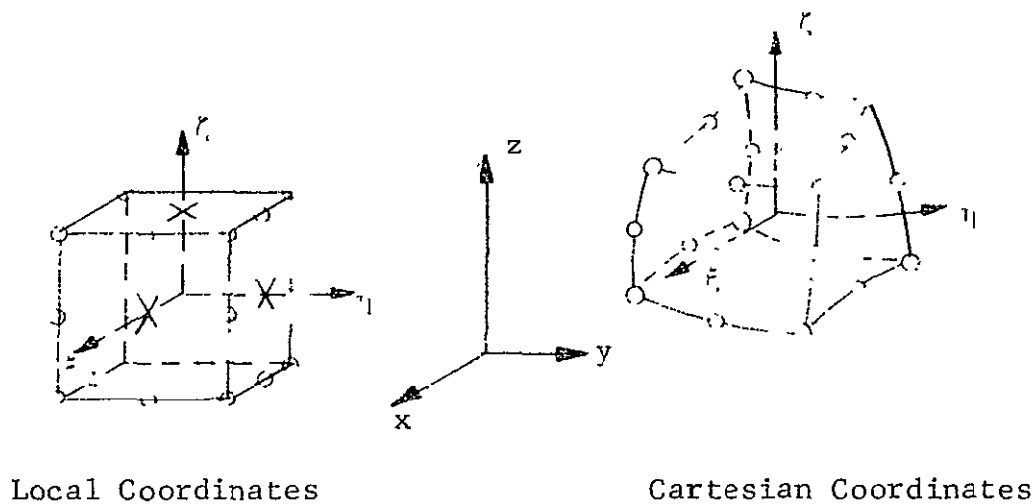


Fig. 1 Isoparametric Hexahedra

NPNTC = 16 Print each element stiffness matrix entry
 — to be stacked with its stacking index
 = 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, $NPNTC = 1 + 32 = 33$.

NPRNT:

If ≤ 0 , perform elastic analysis only.

If > 0 , perform plastic analysis, printing output every NPRNT increments of load.

IRESRT: (See Section XVIII)

- 0, Elastic (and/or) plastic run. Do not generate a restart tape
- 1, Elastic (and/or) plastic (cyclic) run. Build a new restart tape
- 2, Plastic run with elastic values from previously created restart tape
- 3, Plastic restart run starting at some specified load level
- 4, Cyclic restart run

NUTAP:

Applicable only if restarting from a restart tape .

- 0, No new tape written
- 1, Complete new tape created and additional restart data written

INPRT:

- 0, Write restart tape only at $P = P_{MAX}$ (i.e., at maximum load)
- N, Write restart tape every N increments of load

III. Node Specification (1615)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the

program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and $-n$ consecutively is the equivalent of the specification of nodes $m, m+1, m+2, \dots, n$ and specifying $m, -p,$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m+p, m+2p, \dots, m+kp$ where $m+kp$ is the highest integer of the form less than or equal to n . For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Member Connectivity (Node Numbers of Each Member) FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type as follows

- HEX8 - eight node hexahedra element
- HX20 - hexahedra element with up to twelve additional midside nodes
- MSGN - mesh generator for eight-node hexahedra
- SEND - ends the section

The first integer field designates the member number. The next integer field designates the connecting nodes as follows:

- HEX8 - The eight corner nodes are specified in the order shown on Fig. 2.

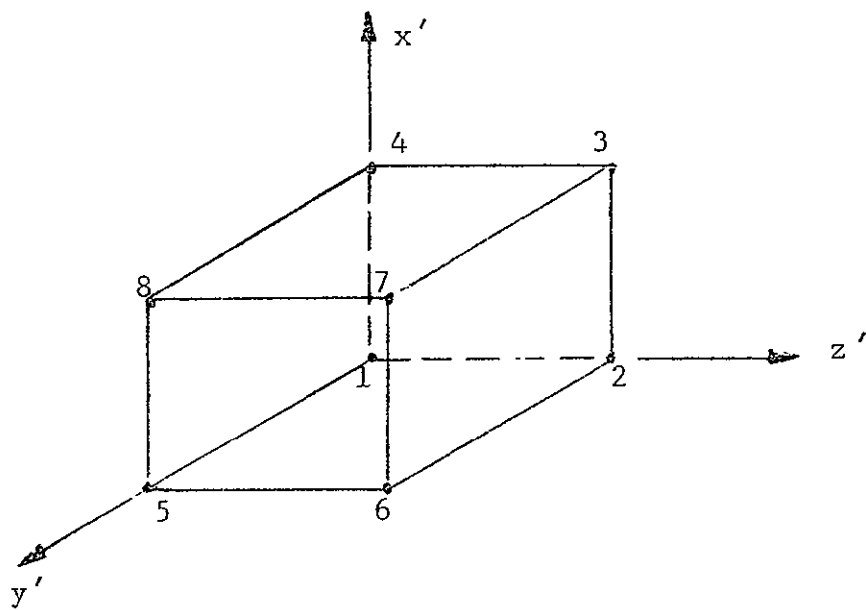


Fig. 2 Order of Corner Nodes

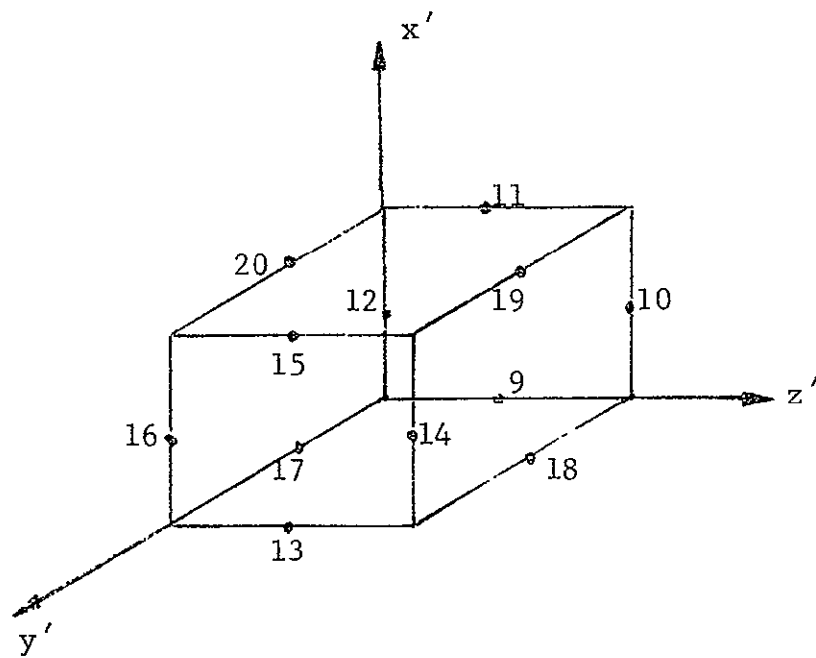


Fig. 3 Order of Midside Nodes

HX20 - The eight corner nodes are specified first in the order shown in Fig. 2. The additional midside nodes are specified on a second card (FORMAT (1215)) according to the order shown in Fig. 3. An element can be constructed with any combination of midside nodes. If a midside node is not specified, a zero is placed in its corresponding position on the card. If a midside node is specified for an element, a midside node should also be specified for every element coincident with that side.

Note: The hexahedra elements with midside nodes can have curved boundaries. The shape of any side is described by a quadratic polynomial through the three node points. If a side containing a midside node is straight, the coordinates of the midside node can be read in as zero. In this case the nodal coordinates are calculated within the program.

MSGN - The nodal connectivity is automatically generated for eight-node hexahedra according to the convention in Fig. 2. The first card for this input option requires the four-character identifier MSGN. A second card is required with FORMAT (5I5) which specifies the number of element subdivisions in three coordinate directions. The integers in the five fields are

the subdivisions in the x_1 , x_2 , x_3 directions, and the first node number and element number in the mesh. The coordinate system must be right handed. The mesh generator assumes that the nodes are numbered consecutively along the x_1 direction specified in the first I5 field then incremented on the $x_1 - x_3$ face until that face is completely specified. The mesh is then incremented in the x_2 direction and then consecutively numbered on the $x_1 - x_3$ plane. The element numbers are numbered consecutively starting at the specified starting value in the positive x_1 direction. The mesh generating capability still requires that the nodes be defined in Section IV and that the nodal coordinates be explicitly defined in Section VI. An example of the convention for a $4 \times 3 \times 2$ mesh in Cartesian coordinates with the starting node and element equal to one is shown in Fig. 4.

V. X-Coordinates of Nodes FORMAT (E15.7, I3I5)

The global x-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given x-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.

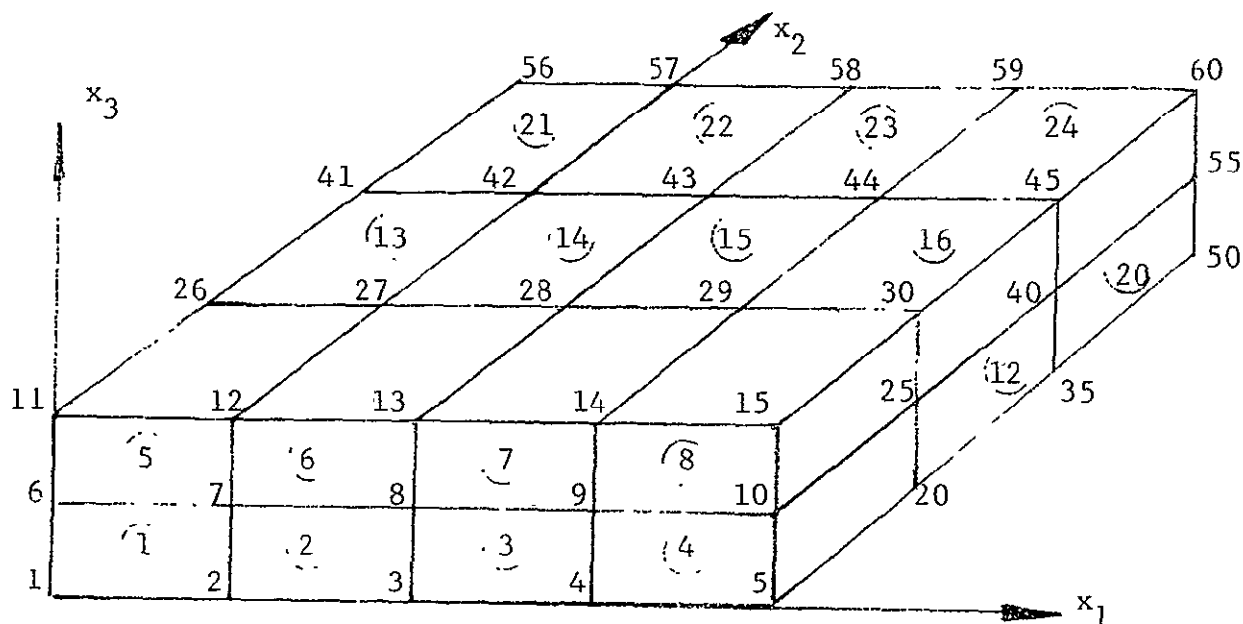


Fig. 4 Node and Member Ordering Convention
for Automatic Mesh Generation

VIII. Boundary Conditions FORMAT (I3, 2I1, 5X, 14I5)

The first three fields give the boundary conditions specifications in the order: u, v, w ; where u, v, w are global displacements in the global x, y, z directions, respectively.

Zero—denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 14I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 14I5 fields are used on a continuation card. A zero or blank I5 field terminates

the card scan for a given boundary condition specification. Note: if the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first 15 field (columns 11-15) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default with all nodes used in the problem specified. Note: Maximum number of nodes is currently 2500.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacement of the form:

$$1) \quad \delta_i = \alpha_1 \delta_1 + \alpha_2 \delta_2 + \cdots + \alpha_n \delta_n$$

where δ_i is a dependent degree of freedom,
 $\delta_1 \cdots \delta_n$ are independent degrees of freedom,
 and $\alpha_1, \alpha_2, \cdots, \alpha_n$ are coefficients.

2) Rotation of displacements at a node

$$\delta_i = \alpha_1 \tilde{\delta}_i + \alpha_2 \tilde{\delta}_j + \alpha_3 \tilde{\delta}_k$$

$$\delta_j = \beta_1 \tilde{\delta}_i + \beta_2 \tilde{\delta}_j + \beta_3 \tilde{\delta}_k$$

$$\delta_k = \gamma_1 \tilde{\delta}_i + \gamma_2 \tilde{\delta}_j + \gamma_3 \tilde{\delta}_k$$

where the α, β, γ 's are the direction cosines of the rotation, $\delta_i, \delta_j, \delta_k$ are the displacements with respect to the original global directions and $\tilde{\delta}_i, \tilde{\delta}_j, \tilde{\delta}_k$ are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

$$\delta_i = \alpha_1$$

where the coefficient α_1 is the applied generalized displacement.

The first two I5 fields designate a node number and a degree of freedom (i.e., 1-3). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material Properties

The input for material properties consists of cards as follows:

CARD 1: FORMAT (A4, 1X, 5E15.7)

MAT1 = four-character identifier

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

ETEE = Young's modulus in principal property axis (3)

VONTO = Poisson's ratio, ν_{12}

VTOTE = Poisson's ratio, ν_{23}

CARD 2: FORMAT (4E15.7)

VTETO = Poisson's ratio, ν_{32}

GONTO = Shear modulus, G_{12}

GTOTE = Shear modulus, G_{23}

GONTE = Shear modulus, G_{13}

CARD 3: FORMAT (3E15.7)

BETA1 = } Angles between 1, 2, 3 material axes
BETA2 = } and local element x, y, z axes (defined
BETA3 = } in the note at the end of the section).

CARD 4: FORMAT (3E15.7)

TALF-1 Coefficient of thermal expansion in
 the principal 1 direction
TALF-2 Coefficient of thermal expansion in
 the principal 2 direction
TALF-3 Coefficient of thermal expansion in
 the principal 3 direction

CARD 5: FORMAT (5E15.7)

SIGOX = Yield stress in principal 1 direction
SIGOY = Yield stress in principal 2 direction
SIGOZ = Yield stress in principal 3 direction
SIGOXY = Shear yield stress in principal
 1-2 plane
SIGOYZ = Shear yield stress in principal
 2-3 plane

CARD 6: FORMAT (5E15.7)

SIGOZX = Shear yield stress in principal
 3-1 direction
RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape
 parameter used in Ramberg-Osgood repre-
 sentation of stress-strain behavior.

 If RMOSS = 0; RMOSN = α the slope of
 the linear strain-hardening stress-strain
 representation, i.e., $\alpha = E_T/E$ where E_T
 is the tangent modulus.

RMOSS = If RMOSS $\neq$ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note (1): If RMOSN = 0 and RMOSS = 0 the material for the element(s) is assumed to be elastic-ideally plastic.

YLDST = Yield stress (for isotropic materials)

Note (2): Only initially isotropic materials can be treated when considering linear or nonlinear strain-hardening.

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

— Note: The transformation between the 1, 2, 3 material axes and the local element x, y, z axes are defined by path dependent angular rotations $\beta_1, \beta_2, \beta_3$. First a rotation about the 1 axis, β_1 , then a rotation about the 2 axis in the transformed system and then a rotation, β_3 about the 3 axis in the transformed system

ACUM - Table describing yield stress versus accumulated plastic strain relationship

FORMAT (A4/A4, 1X, I5, /, A4, 1X, 2E15.7, /, (16I5))

CARD 1:

ACUM - four character identifier

CARD 2:

ACUM - four character identifier

NUMP - number of pairs of points used to represent the yield stress versus accumulated plastic strain

CARD 3:

ACUM - four character identifier

VARSlG - yield stress

SUMEPS - accumulated plastic strain

Card 3 is repeated "NUMP" times where NUMP is the number of pairs in the table. A maximum of 20 pairs is allowed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

GAUSS or LOBATTO - Designation of stress points within an element

CARD 1: FORMAT (A4, 1X, 3I15)

or {
 GAUSS = five letter character identifier
 LOBAT = five letter character identifier
 NX = number of stress points in local x
 direction
 NY = number of stress points in local y
 direction
 NZ = number of stress points in local z
 direction

CARD 2: Applicable members; both shorthands of
 Section III are permitted. Any number of con-
 tinuation cards may be used for a given spe-
 cification. A zero or blank I5 field ends
 each member listing.

SEND in columns 1-4 ends the section.

Note: Up to 8 Gauss or 10 Lobatto points are per-
 mitted in any one direction and a total of
 50 points are permitted in any one element.
 For local coordinate definitions see Figure 2.
 For a definition of these integration points
 see Ref. 7.

XI. Applied Loads

Three different types of loading are currently admissible - concentrated loads at nodes, distributed loads on a surface of a hexahedron member, and thermal loads. Each loading situation is designated by one of the following four-character key words:

CONC (Concentrated load)
SURF (Surface distributed load)
TEMP (Temperatures for thermal loading)

Distributed load -

CARD 1: FORMAT (A4)

Key word SURF

CARD 2: FORMAT (I5, 5X, 8I5)

First I5 field: number of nodes on the loaded surface. There is a minimum of four nodes per surface and a maximum of eight in order to accommodate midside nodes.

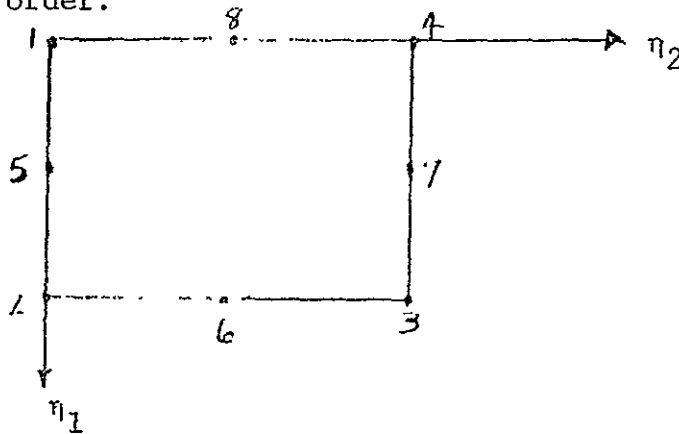
Second through

fifth I5 fields: These four fields contain the four corner node numbers in counterclockwise or clockwise order.

Sixth through

ninth I5 fields: These four fields contain the
midside node numbers (if any).
If a particular midside node
is omitted, a zero or blank is
placed in the appropriate field
on the input card.

The input order for a given surface is shown below for counter-
clockwise order.



The magnitude of the surface tractions at each node specified above
follow: There is one card for each node specified.

SUCCEEDING CARDS: FORMAT (I5, 3E15.7)

NUM = Node number specified on CARD 2 (I5 field)

TX = Surface traction in global x-direction

TY = Surface traction in global y-direction

TZ = Surface traction in global z-direction

A blank card ends the input for surface loads.

Note: The keyword SURF appears only once at the beginning
of the specification of surface loads. Additional
input for surface loads begin with card 2.

Concentrated load -

CARD 1: FORMAT (A4)

Key word CONC

CARD 2: FORMAT (I5, 3E15.7)

NUM = Node number at point of application of
 load (I5 field)

FX = Load component in the global x-direction
 (first E15.7 field)

FY = Load component in the global y-direction
 (second E15.7 field)

FZ = Load component in the global z-direction
 (third E15.7 field)

As many load components as desired may be specified. A zero or blank first I5 field ends the specification of concentrated loads.

Note: The keyword CONC appears only once at the beginning of the specification of concentrated loads. Additional input for concentrated loads begin with card 2.

Thermal load -

Each card after the first designates the temperature increment from some reference temperature followed by its applicable nodes. The input format is identical to that described for the input of nodal coordinates in Section V.

CARD 1: FORMAT (A4, 1X, E15.7)

Key word TEMP

TREF = Reference temperature in E15.7 field

The succeeding cards are as in Section V for nodal coordinates. That is, the nodal temperature in the first E15.7 field followed by the applicable nodes. A blank card ends the specification of nodal temperatures.

Note: The nodal temperature specified at the corner nodes of an individual hexahedra are interpolated to obtain the temperature used in calculating stresses and strains at the stress points.

SEND in columns 1-4 ends the section for applied loading.

XII. Members to be Printed - FORMAT (16I5)

Specify the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A maximum of 2500 members may be specified. Members in excess of 2500 and undefined member numbers are ignored. A blank card or card with only zero entries ends the section.

XIII. Nodes to be Printed - FORMAT (16I5)

Up to 2500 nodes whose displacements are to be printed for the analysis, as per Section XII. A blank card ends the section.

XIV. Parameters for Plastic Analysis - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

XV. Parameters for Succeeding Load Cycles - FORMAT (A5, I5, 2E15.7)

CARD 1: FORMAT (A5)

Keyword defining the load type as follows:

CYCLE = Same load distribution as last half cycle

NEW = New load distribution to be specified

If any other word (or a blank card) is used the analysis is complete.

CARD 2: FORMAT (I5, 2E15.7)

NPRNT = If equal to zero, no additional load cycle and end of problem. If greater than zero, print output every NPRNT increments.

PMAX = New maximum load for this load cycle

PPCT = Load increment as percentage of yield load.

Note: When CYCLE is specified, the original load distribution is maintained. In this case PPCT should be set equal to one-half the original value since the elastic range for subsequent cycles is twice the elastic range for initial loading.

XVa. If NEW is used in Section XV a new load must be specified. This is accomplished with input identical to Section XI - Applied Loads. If CYCLE is specified in Section XV this section is omitted.

XVI. Change Plastic Material Properties When Cycle is Specified

CARD 1: FORMAT (A4, 1X, I5)

MAT1 = Key word

IMEM = One member number from the group that is
to be changed.

CARD 2: FORMAT (5E15.7

SIGOX	}	Defined in Section X
SIGOY		
SIGOZ		
SIGOXY		
SIGOYZ		

CARD 3: FORMAT (5E15.7)

SIGOZX	}	Defined in Section X
RMOSN		
RMOSS		
YLDST		
RMOSE		

The section is ended with a section end card, SEND.

Note: If there is no change in material properties, a SEND card must appear in the appropriate place in the input stream. Succeeding load reversals (CYCLE) or new loads (NEW) are specified with additional input beginning with Section XV. A blank card (or any alphanumeric other than CYCLE or NEW) branches to Section XVII to end the job.

XVII. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVIII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (20A4)

As in Section I

CARD 2: NPNTC, NPRNT, IRESRT, NUTAP, INPRT - FORMAT (5I5)

As in Section II. Here NPNTC is ignored.

If IRESRT is equal to 2

Restart from an elastic critical load using a
previously created restart tape

CARD 3: PMAX, PPCT - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

Input is then continued from Section XV

If IRESRT is equal to 3

Continuation from some previously plastic load. The job can be restarted from a previous maximum load or from any intermediate load level.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form,

* * * * *

PLASTIC ANALYSIS VALUES FOR RESTART TAPE HAVE BEEN
WRITTEN AT P = 1.5000000E+03 FOR PRINT INTERVAL 30

* * * * *

In this case NRSRT is equal to 30 and the load is not necessarily the last maximum value.

Note: For cases when the job ends abnormally (for example when time or lines are exceeded) the job can be restarted with a value for NRSRT obtained from the output. In this case the last value of NRSRT should not be used. This will insure that a complete set of data will be accessed from the restart tape.

If IRESRT is equal to 4

Restart from a previous maximum load level followed by application of a new load (NEW) or reversal of a previous load distribution (CYCLE). A new load or a reversal of a previous load distribution can be placed on the structure from any previously generated and saved state of stress.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSTR is a unique number obtained as output from the job that generated the restart tape. This output has the form shown in the preceding discussion for IRESRT equal to three. The value of PMAX is ignored.

Succeeding cards follow as in Sections XV through XVII.

XIX. Alternate Input Method

All or part of the input may be read from a tape. This tape is mounted as Unit 23 with the appropriate job control cards. The tape is accessed by specifying the keyword, TAPE (FORMAT A4), in the proper place in the card input stream. Input will then be read from the tape until a SEND (A4) is encountered on the tape. This returns the input reader to the card input stream.

Example:

Input for member connectivity, Section IV, and coordinates have been generated on a tape in the prescribed manner.

Input cards are as follows:

TITLE

0 5

1 - 100

TAPE (reads data from a tape)

```

HEX8
.      (additional elements if desired)
.
.
SEND

TAPE      (reads  X-coordinates from tape)
          (Blank card to end section for X-coordinates)
TAPE      (reads  Y-coordinates from tape)
          (Blank card to end section for Y-coordinates)
5.0      (card input for  Z-coordinates
.
.
.
          (Blank card ends section for  Z-coordinates)

```

If no additional tape input, remaining card input as usual.

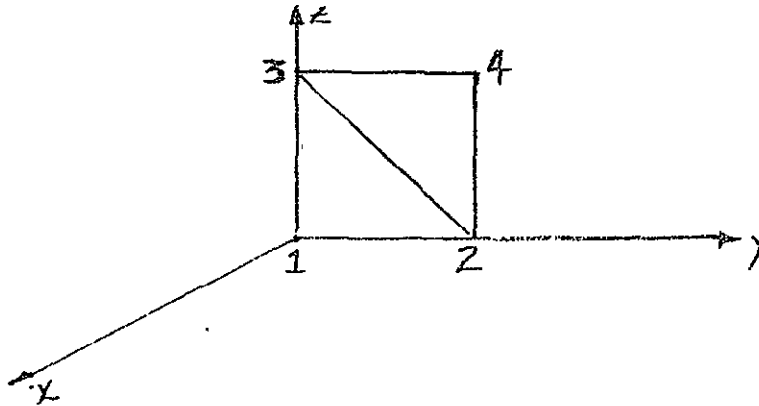
Note: All SEND formats used to end sections must be in card form as the SEND encountered on the tape only sends the reader back to the card input stream. Also blank cards which end sections are best not put on the tape but rather on cards as this allows the user to add card input to tape input, i.e., if a blank ends a section and is encountered on the tape format then the section ends and no new data may be added, but if the blank does not occur on tape the section is still open and can be closed with a blank card in the card input stream.

REFERENCES

1. Zienkiewicz, O, et al., "Iso-Parametric and Associated Element Families for Two and Three Dimensional Analysis," Finite Element Methods in Stress Analysis, TAPIR, Technical University of Norway, Trondheim, 1969.
2. Levy, A., "A Three Dimensional 'Variable Node' Isoparametric Solid Element," Grumman Research Department Memorandum RM-587, July 1974.
3. Armen, H., Levine, H., Pifko, A., and Levy, A., "Nonlinear Analysis of Structures," NASA Contractors Report CR-2351, March 1974; also Grumman Research Department Report RE-454, May 1973.
4. Prager, W., "The Theory of Plasticity: A Survey of Recent Achievements," (James Clayton Lecture) Proc. Inst. Mech. Engrs., Vol. 169, p. 31, 1955.
5. Ziegler, H., "A Modification of Prager's Hardening Rule," Quart. Appl. Math., Vol. 17, No. 1, p. 55, 1959.
6. Armen, H., Jr., Pifko, A., and Levine, H., "Finite Element Analysis of Structures in the Plastic Range," NASA Contractor Report CR-1649, February 1971.
7. Abramowitz, M. and Stegun, I. A., Eds., Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series, 1964.

APPENDIX I — MULTIPOINT CONSTRAINTS

EXAMPLE 1



Node 1 and 2 are fixed.

There is an applied displacement at node 3 in the negative y direction. The distance between nodes 3 and 4 remains constant.

Boundary Conditions

011000 1-4 default condition

001000 1 2 4

021000 — 3

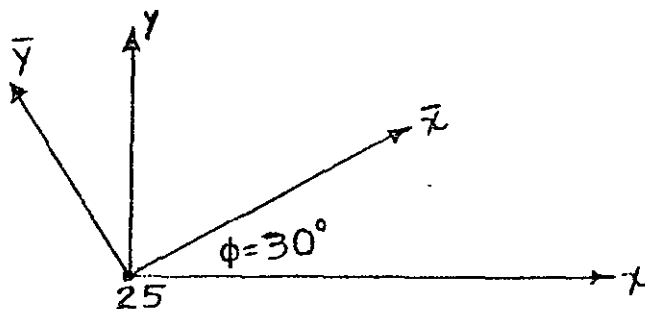
Blank card

Dependencies

Node	Dof	Node	Dof		
3	2	3	2	-.050	(specified displacement)
4	2	3	2	1.0	restraint condition that
4	3	3	3	1.0	3-4 remain rigid

Note: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).

EXAMPLE 2 Rotation of Displacements at a Boundary



For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are x, y . Thus:

$$u = \bar{u} \cos \phi - \bar{v} \sin \phi$$

$$v = \bar{u} \sin \phi + \bar{v} \cos \phi$$

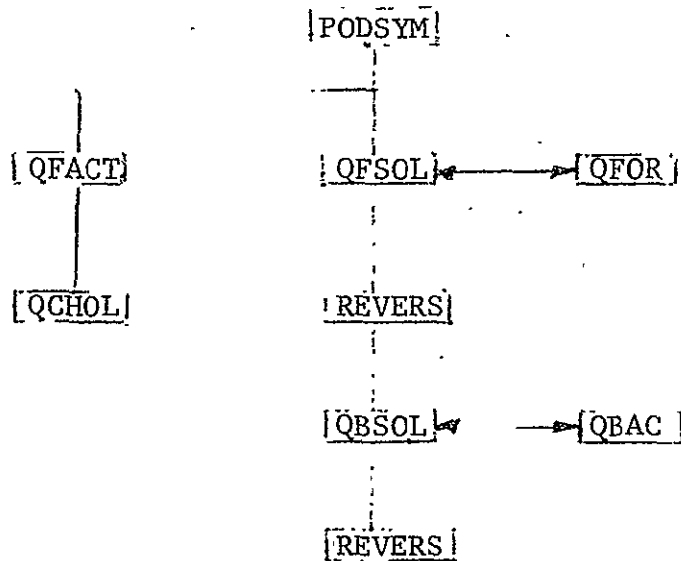
The multipoint constraint cards are:

25	1	25	1	.86602540E00	25	2	-.50E00
25	2	25	1	.50E00	25	2	.8660254E00

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.

APPENDIX II — SOLUTION PACKAGE

PODSYM - Solution of Symmetric Positive Definite Banded Matrix Equations



Problem

Solve $AX = \underline{Y}$, where

$$A = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

is a banded positive definite symmetric matrix; X is the desired solution vector; and Y is the known right side (load vector). PODSYM is the user interface and supervisory routine. It uses the Cholesky algorithm which factors the total stiffness matrix into LL^T (where L is a lower triangular matrix) and then solves a pair of triangular sets of equations.

The factorization is carried out by subroutine QFACT which supervised the storage and data set allocation and subroutine QCHOL which generates the lower triangular L matrix. QCHOL implements the Cholesky algorithm to factor the positive definite symmetric A matrix as the product of a lower triangular matrix with its transpose:

$$A = LL^T$$

A straightforward argument establishes the possibility of decomposing any positive definite matrix in this fashion. Once L has been obtained, it is not difficult to solve the system of linear equations $AX = Y$ by calculating Z as the solution of the lower triangular system $LZ = Y$, and then X as the solution of the upper triangular system $L^T X = Z$. The forward solution ($LZ = Y$) is accomplished by subroutines QFSOL and QFOR and the back solution ($L^T X = Z$) by subroutines QBSOL and QBAC. Before the call to QBSOL, subroutine REVERS is called, which reverses the rows of L so that the last row becomes the first row, etc. This is accomplished in order to sequentially access L^T during the back solution.

The above algorithm is noteworthy for its assured stability and general efficiency. It is possible to carry out an error analysis of the procedure as it is represented on a digital computer; such analysis shows that the computed L matrix satisfies an equation of the form

$$A + E = LL^T$$

with bounds on the elements of E which show that E is small compared to A. The effect of rounding errors made in the subsequent solution of $LZ = Y$ and $L^T X = Z$ may then be taken into

account by (implicitly) introducing an additional perturbation F into A , and it is then concluded that the computed solution X_0 exactly satisfies the equation

$$(A = E + F)X_0 = Y$$

Since $E + F$ can be shown to be small, one would like to infer that X_0 is almost an exact solution of the original equations, and unless A is too nearly singular, such a conclusion is indeed warranted. But if A is very ill-conditioned, no such result can be guaranteed, and X_0 may be far from the mathematically correct solution; in this event single-precision computation will not suffice for the calculation of an accurate solution, and since the solution will be very sensitive to small errors in A , it is unlikely that even a high-precision computation will yield satisfactory results unless A and Y are known (and supplied) to more than single-precision accuracy. The PODSYM subroutines make a fairly realistic attempt to detect and report pathological conditions of this sort.

The large positive definite matrices that occur in practical work very often contain a large number of zero entries and the program seeks to benefit from the presence of these elements by modifying the standard Cholesky formulae

$$l_{kk} = \left[A_{kk} - \sum_{j=1}^{k-1} l_{kj}^2 \right]^{1/2}$$

and

$$l_{ik} = \left[A_{ik} - \sum_{j=1}^{k-1} l_{ij} l_{kj} \right] / l_{kk} \quad \text{for } i > k$$

to read instead

$$l_{kk} = \left[A_{kk} - \sum_{j=v(k)}^{k-1} l_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$l_{ik} = \left[A_{ik} - \sum_{j=\mu(i,k)}^{k-1} l_{ij} l_{kj} \right] / l_{kk}$$

INSTRUCTIONS FOR
THE USE OF REVBY

A Program for the Elastic-Plastic
Analysis of Bodies of Revolution

INSTRUCTIONS FOR THE USE OF REVBY

REVBY is a finite element program to treat the elastic, elastic-plastic, or elastic-cyclic plastic response of orthotropic axisymmetric solids of revolution under axisymmetric loadings. Currently, three different types of elements are available for the analysis of thick and thin bodies of revolution:

1. Revolved Triangular Element

The revolved triangular element used is similar to the element presented in Ref. 1, restricted to axisymmetric loading, i.e., independent of the circumferential direction, but including orthotropic material properties (see Fig. 1).

2. Thin Shell Element

The thin shell element used was developed at Grumman and the theoretical derivations are presented in Refs. 2 and 3. Cubic Hermitian polynomials represent the three displacement components. Again, only axisymmetric loading is allowed but orthotropic material properties are included (see Fig. 2).

3. Thin Ring Stiffener Element

The thin ring stiffener element is similar to the one presented in Ref. 4 with the exception that the ring may be attached with arbitrary eccentricities. No warping of the cross section is allowed and the shear center and centroid must coincide. Five different cross-sectional geometries are available.

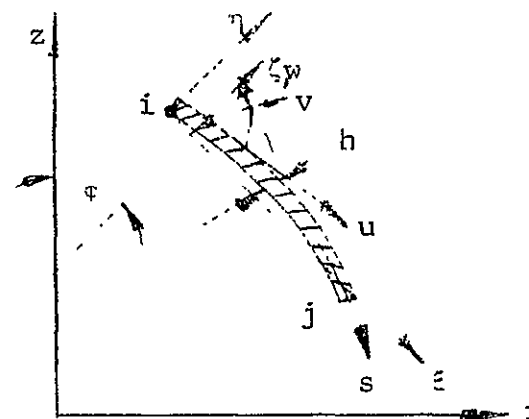
$$u_r = a_1 + a_2 r + a_3 z$$

$$u_z = a_4 + a_5 r + a_6 z$$

$$u_3 = a_7 + a_8 r + a_9 z$$

$$\epsilon_{ij}(r,z) = \text{constant}$$

Fig. 1 Axisymmetric Revolved Triangle


$$u = a_1 + a_2 \xi + a_3 \xi^2 + a_4 \xi^3$$

$$w = b_1 + b_2 \xi + b_3 \xi^2 + b_4 \xi^3$$

$$v = c_1 + c_2 \xi + c_3 \xi^2 + c_4 \xi^3$$

$$\begin{Bmatrix} \epsilon_s \\ \epsilon_\theta \\ \gamma_{s\theta} \end{Bmatrix} = (1 - \xi) \begin{Bmatrix} \epsilon_s^i(\xi) \\ \epsilon_\theta^i(\xi) \\ \gamma_{s\theta}^i(\xi) \end{Bmatrix} + \xi \begin{Bmatrix} \epsilon_s^j(\xi) \\ \epsilon_\theta^j(\xi) \\ \gamma_{s\theta}^j(\xi) \end{Bmatrix}$$

Fig: 2 Axisymmetric Thin Shell Element

The plasticity theory used is presented in Refs. 5, 6, and 7. The input to the program is categorized in the following sections.

I. Problem Title (20A4)

Any 80-character title describing the problem.

II. NPNTC and NPRNT (215)

NPNTC: is for selective printing. NPNTC is the sum of the following integers corresponding to the options desired.

If NPNTC = 0, no intermediate output
= 1, print total load vector
= 8, print nonzero element stiffness matrix entries with their stacking indices
= 16, print the total stiffness matrix

NPRNT: If ≤ 0 , perform elastic analysis only
If > 0 , perform elastic and plastic analysis, printing the results of the plastic analysis every NPRNT load increments

III. Nodes by Partition (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the

program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and $-n$ consecutively is the equivalent of the specification of nodes $m, m+1, m+2, \dots, n$ and specifying $m, -p,$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m+p, m+2p, \dots, m+kp$ where $m+kp$ is the highest integer of the form less than or equal to n . For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Node Numbers for Members (A4,6X,4I5)

The first field defines the element type:

TRIR - triangular element designation (Ref. 1)

SHEL - shell element designation (Refs. 2, 3)

RING - ring element designation (Ref. 4)

The first integer field designates a member number and the three following fields give node numbers for the member.

For triangular members, three nodes are specified. The i, j, k nodes are given in counterclockwise order. For shell members, two nodes are specified, i and j , in the positive s direction. For ring members, only one node is specified. SEND in the first field ends the section.

Note: Maximum number of elements (members) allowed is 600 currently.

V. Global Coordinates (E15.7,13I5)

The coordinates are read in a specified order, r first, then z , and last the PHI coordinates. PHI is the angle in radians between the z -axis and the normal to the shell surface, measured positive clockwise (Fig. 2) from the z -axis. The floating point number designates a coordinate value. Following are the applicable node numbers as in Section III.

Only the integer fields are used on continuation cards. A zero or blank node field other than the first field on a noncontinuation card ends the card scan. On a continuation card, the floating field is left blank. A zero or blank first node field on a noncontinuation card ends each section for r , z , and PHI. If there are no shell elements, PHI is set to zero by default and need not be specified.

VI. Boundary Conditions (6I1,4X,14I5)

The first six fields give a boundary condition specification, in the order:

- 1) u_r, u_z, u_θ , if the node is attached to a triangular member. The last three fields for such a node must be zero.
- 2) $u, w, \chi, \epsilon_s, v, \gamma_{s\theta}$, if the node is attached to a shell element. See Fig. 2 for convention. u is the meridional displacement of the shell middle surface, w is the normal displacement, χ is the rotation of a normal to the middle surface, ϵ_s is the meridional membrane strain, v is the circumferential displacement, and $\gamma_{s\theta}$ is the shear strain. At a pole, the condition $u_r = 0, \chi = 0, v = 0, \gamma_{s\theta} = 0$, must be prescribed.

3) u, w, χ if the node is attached to a ring member.
The last three fields must be zero.

Zero - denotes a fixed degree of freedom

One - denotes a free degree of freedom

Two - denotes an applied displacement and a corresponding card must be included in Section VII, designating the magnitude of the applied displacement.

The integer fields are the applicable node numbers as in Section III. On continuation cards, only the integer fields are used. If a node's boundary conditions are not specified in this section, all six degrees of freedom are assumed free. To change the default, the first card should be set to the desired default (i.e., for triangles 111000) with all nodes used in the problem then specified.

Blank card ends section.

Note: Maximum number of nodes currently allowed is 900.

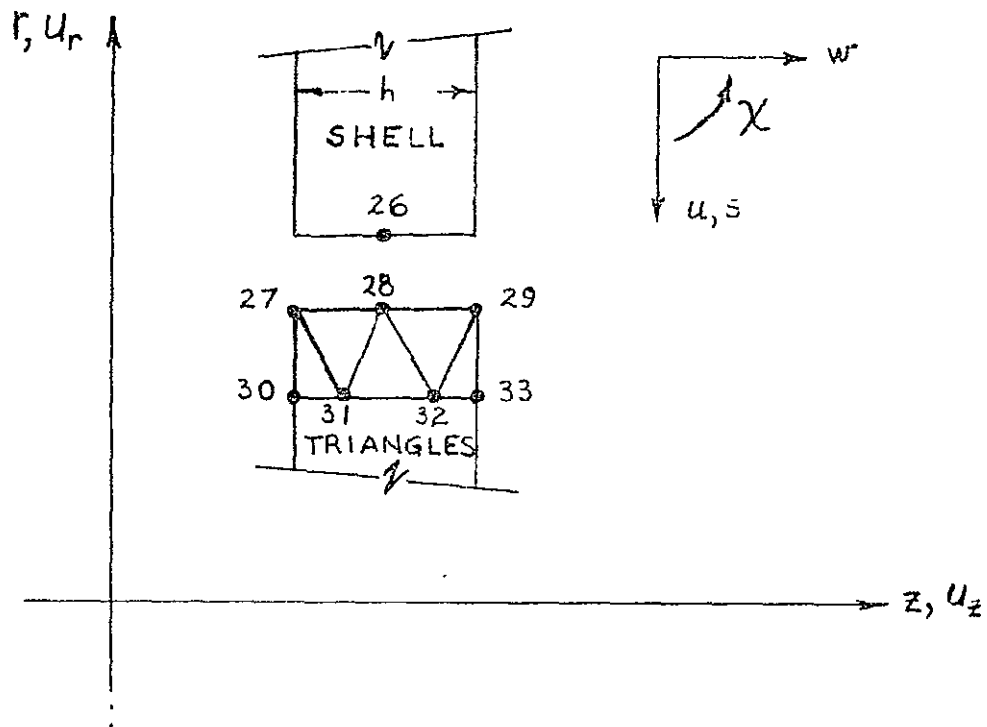
VII. Dependent Degrees of Freedom (2I5,2(5X,2I5,E15.7))

The first two fields designate a node number and its degree of freedom. The dependency is defined in the following three fields. The two integer fields are node number and degree of freedom. The coefficient is contained in the following field. If there is another dependency for the node, it would be designated in the last three fields.

If there is yet another dependency, it would appear on a continuation card with the first two integer fields blank. Both single and multipoint constraints of the form $\delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \dots + \alpha_n \delta_{jn}$ may be considered as well as applied displacements of the form $\delta_i = \alpha_1$, e.g.,

Blank card ends section.

1. Linking Up a Shell and a Triangle for a Cylinder



From shell theory restrictions

$$u_{r27} = u_{r28} = u_{r29} = w_{26}$$

$$u_{z27} = -u_{26} - h/2 \chi_{26}$$

$$u_{z28} = -u_{26}$$

$$u_{z29} = -u_{26} + h/2 \chi_{26}$$

The input cards, for this case, would appear as (let $h = 0.1$ inches)

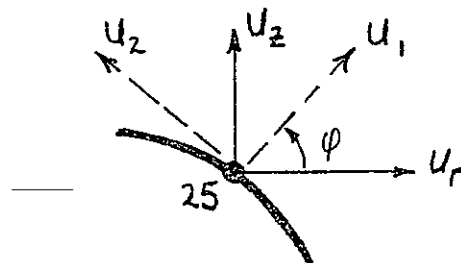
```

27  1  26  2  1.0 E00
28  1  26  2  1.0 E00
29  1  26  2  1.0 E00
27  2  26  1 -1.0 E00  26  3  - 0.05 E00
28  2  26  1 -1.0 E00
29  2  26  1 -1.0 E00  26  3  + 0.05 E00

```

Note: In the boundary condition cards for nodes 27, 28, and 29, degrees of freedom 1 and 2 must be fixed (specified = 0) because they are effectively eliminated from the solution (they are dependent degrees of freedom).

2. Rotation of Displacements at Boundary



For this problem, we set the displacement normal to the boundary equal to zero. We have:

$$u_1 = u_r \cos \phi + u_z \sin \phi$$

$$u_2 = -u_r \sin \phi + u_z \cos \phi$$

Let us assume that the node in question is node 25 and $\varphi = 30$ degrees. The multipoint constraint cards are:

```

25  1  25  1    0.86602540 E00  25  2  0.50      E00
25  2  25  1   - 0.50      E00  25  2  0.86602540 E00

```

Note: On the boundary condition cards, since $u_1 = 0$, node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 (or free) since it remains an independent degree of freedom (although rotated). If the normal displacement u_1 were not fixed (and the transformation performed merely to apply a normal load) it would have a 1 boundary condition specified.

3. Applied Displacement

Suppose for a shell we wish to apply a unit lateral (w) displacement at node 25. We have the multipoint constraint card as

```

——— 25  2  25  2  1.0 E00

```

Note: A 2-boundary condition for node 25 degree of freedom 2 is specified on the boundary condition card. The end of this section is designated by a blank or zero in the third integer field (e.g., blank card).

VIII. Material Properties

The format of this input is dependent upon the member type:

A. Triangle Elements (4(5E15.7),3E15 7,/, (16I5))

Card 1	E1	Young's modulus in 1 direction (if $\beta = 0$, θ -direction)
	E2	Young's modulus in 2 direction (if $\beta = 0$, z-direction)
	E3	Young's modulus in 3 direction (if $\beta = 0$, r-direction)
	V12	Poisson's ratio in 1-3 directions
	V23	Poisson's ratio in 2-3 directions
Card 2	V31	Poisson's ratio in 3-1 directions
	G12	Shear modulus in 1-2 plane
	G23	Shear modulus in 2-3 plane
	G13	Shear modulus in 3-1 plane
	β	Orientation of material properties axis with respect to θ -z coordinates (in degrees). This is a rotation about the r-axis and β is positive clockwise.

Card 3	SIGOX	Tensile (or compressive) yield stress in θ -direction
	SIGOY	Tensile (or compressive) yield stress in z -direction
	SIGOZ	Tensile (or compressive) yield stress in r -direction
	SIGXY	Shear yield stress in r - θ plane
	RMOSN	Ramberg-Osgood shape parameter (n), $\epsilon = \sigma/E + (3\sigma/7E)(\sigma/\sigma_{0.7})^{n-1}$. If set = 0.0 material for the group of members is assumed to elastic ideally plastic. If RMOSS = 0.0 and RMOSN $\neq$ 0, RMOSN is ratio of slope of stress- strain curve for linear strain hardening to Young's modulus ($\alpha = E_T/E$ where E_T is the tangent modulus).
Card 4	RMOSS	Ramberg-Osgood parameter $\sigma_{0.7}$. If zero and RMOSN not zero have linear strain hardening.
	YLDST	Yield stress in tension (or compression)
	RMOSE	Ramberg-Osgood parameter E (Young's modulus)
	SIGYZ	Shear yield stress in θ - z plane
Card 5	SIGZX	Shear yield stress in z - r plane
	TALPH (1)	3 thermal coefficients of expansion in 1, 2, 3 directions (θ, z, r if $\beta = 0^\circ$)
	TALPH (2)	
	TALPH (3)	

Note: Orthotropic material properties should not be used for
strain hardening problems in the current version of REVBY.

Succeeding cards give applicable members as in Section III.
A zero or blank E1 ends the section. If there are no triangular members, no card is required.

B. Shell Elements ((4E15.7,/,5E15.7,/,3E15.7,/,2E15.7)/),(16I5))

1) Material Properties

Card 1	E1	Young's modulus in meridional direction
	E2	Young's modulus in circumferential direction
	G33	Shear modulus in plane of shell median surface
	V12	Poisson's ratio in plane of shell median surface
Card 2	SIGOX	Tensile or compressive yield stress in meridional direction
	SIGOY	Tensile or compressive yield stress in circumferential direction
	SIGOZ	Tensile or compressive yield stress in direction normal to shell surface
	SIGXY	Shear yield stress in plane of shell median surface
	RMOSN	(See Card 3 for triangular members)
Card 3	RMOSS	} (See Card 4 for triangular members)
	YLDST	
	RMOSE	
Card 4	TALPH (1)	} Two thermal coefficients of expansion in meridional and circumferential direction, respectively
	TALPH (2)	

Succeeding cards give applicable members as in Section III. A zero or blank E1 ends the section. If there are no shell elements, no blank card is required.

- 2) Thickness for Shell Elements (follows the material properties. All nodes must have a thickness specified.)

(E15.7,/(16I5))

First field designates thickness value. Applicable nodes follow as in Section III. Section ends with zero or blank in first field.

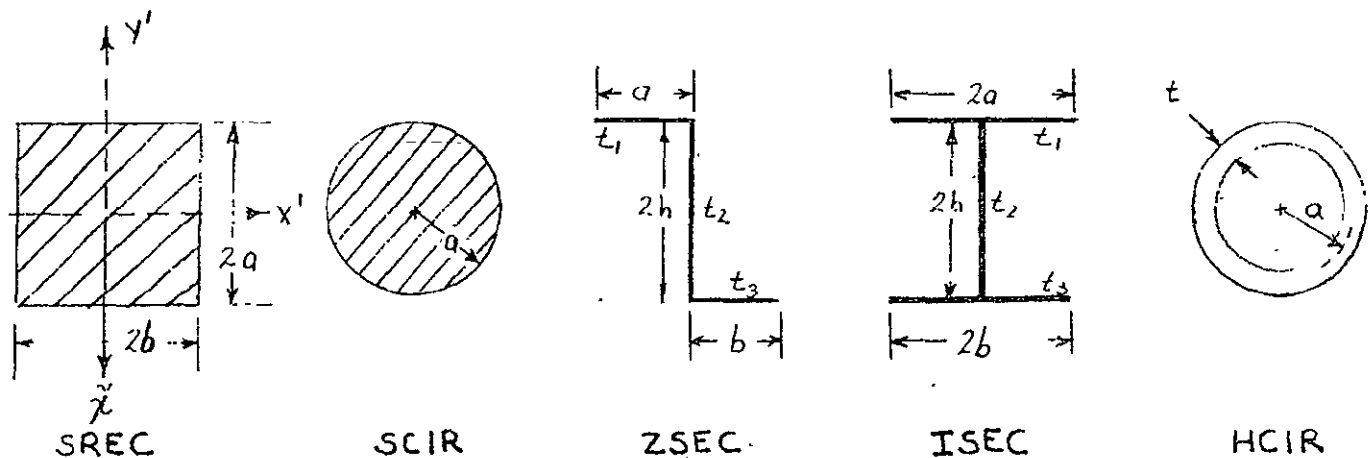
- 3) NG and NLRS (2I5). For shell elements, one card. NG designates the number of Gauss points used in integration scheme for shell element, and the second field, NLRS, designates number of layers through the thickness at which plastic strains, stresses, etc., are calculated. $NG \leq 8$, NLRS must be even, and ≤ 30 .

C. Ring Elements (5E15.7,/,2E15.7,/,A4.6X,3E15.7,/,3E15.7,/,3E15.7,/,E15.7,(16I5))

Card 1	E	Young's modulus
	A	Cross sectional area of ring
	IX	Moment of inertia of ring about centroidal x'-axis (see diagram below)
	IXY	Product of inertia of ring about centroidal axis
	IY	Moment of inertia of ring about centroidal y'-axis

Card 2	Z0	Distance from shell middle surface to ring centroid (normal to shell middle surface). Plus in negative z-direction.
	ECCEN(Y)	Distance along meridian from ring centroid to point of attachment to shell middle surface. Plus in negative s-direction.
Card 3	SREC	Solid rectangular cross section
	SCIR	Solid circular cross section
	ZSEC	Z-section
	ISEC	I-section
	HCIR	Hollow circular cross section

Remaining three fields designate section dimensions as shown in the diagrams below.



INPUT

	SREC	SCIR	ZSEC	ISEC	HCIR
Card 3	2a	a	a	2a	a
	2b		2h	2h	t
			b	b	
Card 4			t_1	t_1	
			t_2	t_2	
			t_3	t_3	

Note: For ZSEC, $a = b$, $t_1 = t_3$; for ISEC, $2a = 2b$, $t_1 = t_3$.

Card 4 If section name is ZSEC or ISEC, this card is required for three additional dimensions, otherwise this card is omitted.

Card 5 SIGOX Yield stress in tension or compression
 RMOSN } Same as shell and/or triangle
 RMOSS }

Card 6 TALPH (1) Thermal coefficient of expansion

Succeeding cards give applicable members as in Section III.
 A zero or blank E ends the section. If there are no ring elements, no card is required.

IX. Load Vector Components

This format is also dependent on member type.

A. Triangular Members (I5,3E15.7,/,I10,3E15.7)

The integer field gives node number (i-j pairs); the floating-point fields give components at the respective nodes as follows:

$p_r = 1.0$ corresponds to an applied unit pressure
(force/unit area) in the radial direction
at node i

$p_z = 1.0$ corresponds to an applied unit pressure
(force/unit area) in the z-direction
at node i

$p_\theta = 1.0$ corresponds to an applied unit pressure
(force/unit area) in the θ -direction
at node i

The load between any two adjacent nodes is assumed to vary linearly. Therefore, the load vector is specified by two cards, the first specifying the load at node i and the second, the load at node j. A zero or blank first integer field ends the section. If there are no uniform loads, a blank card is required. If there are no triangular elements, no card is required.

B. Shell Members (3E15.7,/, (16I5))

The three fields designate the meridional component, p_s , normal component, p_z , and the circumferential component, p_θ , of applied stresses at nodes. Applied stresses are assumed to vary linearly from node to node. The applicable nodes follow as in Section III. A blank card ends the section. If there are no uniform loads, a blank card is required. If there are no shell elements, no card is required.

C. Concentrated Loads (4E15.7,/, (16I5))

For triangle component, specify F_r , F_z , and F_θ in r , z , θ directions, and for the shell component specify N_s , Q_s , M_s , and $N_{s\theta}$ in u , w , χ , and v directions, respectively, in lb/in. These are forces per unit length of circumference, except at $r = 0$, where $F_r = F_\theta = 0$ and $N_s = M_s = N_{s\theta} = 0$ and F_z or Q_s represent an actual force (lbs).

Applicable nodes follow as in Section III. A blank or zero card ends the concentrated loads specification section.

X. Nodal Temperatures (A4,E15.7,/(A4,3E15.7,/, (16I5))

If the first field is SEND, there are no temperatures and this section ends. If the first field is blank, there are thermal loads and the floating number field designates the reference temperature value (T_{ref}), which is only for the user's information.

The next cards contain $\Delta T = T - T_{ref}$ at each node (up to three per node depending on the element to which it is attached) in the floating point fields. For a triangular element, one temperature is specified per node. For a shell element, three temperatures are specified, one at the top surface ($+h/2$), T_1 , one at the median surface ($z = 0$), T_2 , and one at the bottom surface ($-h/2$), T_3 . The program assumes a parabolic distribution of temperature through the thickness (if $T_2 = (T_1 + T_3)/2$, this reduces to a linear variation). For a ring element, three temperatures are specified. A parabolic distribution is assumed through the ring depth. T_1 is the temperature at $\tilde{x} = +a$ or $+h$. T_2 is the temperature at the centroid, and T_3 is the temperature at $\tilde{x} = -a$ or $-h$ (see diagram in Section VIII).

Applicable nodes follow as in Section III.

XI. Members Whose Results are to be Printed for Elastic Solution (1615) As in Section III

Blank cards end section.

XII. Nodes Whose Results are to be Printed for Elastic Solution (1615) As in Section III

Blank cards end section.

XIII. Same as Section XI for Plasticity, if applicable.

Blank cards end section.

XIV. Same as Section XII for Plasticity, if applicable.

Blank cards end section.

XV. PMAX and PPCT (2E15.7)

PMAX - Maximum load to be applied for this half cycle

PPCT - Load increment as a percentage of yield load

XVI. NPRNT, PMAX, PPCT (15,2E15.7)

For succeeding load cycles, one card giving new NPRNT, PMAX, and PPCT. Zero NPRNT signifies no new load cycle and end of problem. For succeeding half-cycles, PPCT equal to one-half the original value should be used since the elastic range is twice the elastic range for initial loading.

XVII. Change in Material Properties (A4)

To change properties of any group of members, read indicator for new material properties (A4).

- A. If RING is input, it indicates a change in the RING element material properties. A card with a member number belonging to the group whose properties are to be changed and the new quantities are read in (15,4E15.7).

Member Number	Member of group to be changed
SIGOX	
RMOSN	New Values
RMOSS	
TALPH	

B. NEWM signifies new material properties for either triangle or shell elements. The new values are read in (I5,5E15.7,/,5E15.7,/,3E15.7)

	Member Number	Member of group to be changed
	SIGOX	
	SIGOX	
Card 1	SIGOZ	New Values
	SIGXY	
	RMOSN	

	RMOSS	
	YLDST	
Card 2	RMOSE	New Values
	SIGYZ	
	SIGZX	

	TALPH (1)	
Card 3	TALPH (2)	New Values
	TALPH (3)	

C. SEND as input indicates no new material properties or end of this section.

XVIII. Each Problem's Input Must Be Ended with a Card Reading "ENDb" Where "b" Denotes a Blank, in Columns 1-4. The Last Problem in a Run Should End with a Card Reading "STOP" in Column 1-4 Instead of "ENDb."

References

1. Wilson, E. L., "Structural Analysis of Axisymmetric Solids," AIAA Journal, Vol. 3, pp. 2269-2274, 1965.
2. Levine, H. S. and Armen, H., Jr., "A Refined Doubly-Curved Axisymmetric Orthotropic Shell Element," Grumman Research Department Memorandum RM-496, February 1971.
3. Levine, H. S., Armen, H., Jr., Winter, R., and Pifko, A., "Nonlinear Behavior of Shells of Revolution Under Cyclic Loading," Grumman Research Department Report RE-426J, April 1972; also presented at the National Symposium on Computerized Structural Analysis and Design, Washington, D.C., March 27-29, 1972; to appear in Journal of Computers and Structures.
4. Cohen, G., "Computer Analysis of Asymmetric Buckling of Ring-Stiffened Orthotropic Shells of Revolution," AIAA Journal, Vol. 6, No. 1, 1968.
5. Isakson, G., Armen, H., Jr., and Pifko, A., "Discrete Elements Methods for the Plastic Analysis of Structures," NASA CR-803, 1967.
6. Armen, H., Jr., Pifko, A., and Levine, H. S., "Finite Element Analysis of Structures in the Plastic Range, NASA CR-1649, 1971.
7. Armen, H., Jr., Levine, H., and Pifko, A., "Plasticity-Theory and Finite Element Applications," Grumman Research Department Report RE-438J, August 1972; also presented at the Second Japanese-U.S. Conference on Matrix Methods, Berkeley, California, August 14-18, 1972; published in Advances in Computational Methods in Structural Mechanics and Design, University of Alabama Press, Huntsville, Alabama.

INSTRUCTIONS FOR USE OF
OUT-OF-PLANE GM

A Program for the Nonlinear
Analysis of Built-Up Structures

INSTRUCTIONS FOR USE OF OUT-OF-PLANE GM

OUT-OF-PLANE GM is a finite element program for the nonlinear analysis of arbitrary built-up thin walled structures. This program considers the sheet material to be in states of membrane stress. The finite element library consists of the following elements:

- Three-node uniform stress triangle
- Six-node linearly varying stress triangle
- Four- and five-node hybrid triangles to be used as transition elements between three- and six-node elements (see Fig. 1a)
- Two-node uniform stress stringer
- Three-node linearly varying stress stringer
- Beam with various cross sections subjected to bending about two planes as well as torsion

The program is capable of treating the elastic and the elastic-ideally plastic response of orthotropic materials. In addition, consideration is given to isotropic materials exhibiting elastic-ideal plastic, linear strain hardening, or nonlinear strain hardening behavior. Further, the kinematic hardening theory of plasticity is used.

The effects of geometric nonlinear behavior is treated by considering a convected coordinate system that accounts for changes in geometry and by the introduction of an initial stress stiffness matrix. The solution strategy implemented by the program is outlined in a companion volume, "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractor Report NASA CR-2568, p. 31.

The input to the program is categorized in the following sections.

I: Problem Title FORMAT (20A4)

Any 80-character title describing the problem.

II. Load and Print Control Parameters FORMAT (6I5./4E15.7)

CARD 1:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0 No intermediate printout
 = 1 Print the load vector
 = 2 Print element stiffness matrix
 = 16 Print each element stiffness matrix entry to be
 stacked with its stacking index
 = 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, $NPNTC = 1 + 32 = 33$.

NPRNT $\geq$ 0 Print output every NPRNT increments of load.
 If NPRNT equals zero the output is printed
 every load step.

NFORM $\geq$ 0 Reform elastic stiffness matrix to account for
 changing geometry every NFORM increments of load.
 The effect of the geometric (initial stress)
 stiffness matrix is still introduced as an "ef-
 fective load" in this case.

IREST: IREST = 0 Do not generate a restart tape
 IREST = 1 Build a restart tape
 IREST = 2 Run continuing from a created
 restart tape

NUTAP: Applicable only if restarting from a restart tape. When IREST = 2 and NUTAP = 0 no additional restart data is written. If NUTAP $\neq$ 0 restart tape is continued.

INPRT: INPRT = 0 Write restart tape only at P = PMAX
(i.e., at maximum load)
INPRT = N Restart data written every INPRT time steps

NRSRT: Applicable only if restarting from a restart tape. NRSRT is a unique number that is printed in the job that generated the restart tape for each load that data is written on the restart tape. This number defines the starting load for the continuation job (see Section XVII).

CARD 2:

PMAX: Maximum load factor. The load factor multiplies the vector of applied load. If the applied loading is normalized to unity then PMAX represents the magnitude of the largest component of the applied load. If the maximum desired load is input then PMAX is equal to one and each load step represents a percentage of the total load.

DELP: Number of load increments. The load increment ΔP is specified by $PMAX/DELP$.

PTAN: Load at which full tangent modulus method is used, i.e., the stiffness matrix explicitly includes the initial stress stiffness matrix. If the "tangent modulus" method is to be used for the entire analysis, PTAN = 0.0. In this case the stiffness matrix is reformed in every load step. If the "effective

load" method is to be used for the entire analysis, $PTAN = PMAX$. In this case the stiffness matrix includes only the elastic stiffness matrix and is reformed every NFORM load steps. When $0 < PTAN < PMAX$ then the program uses the "effective load" method up to PTAN and then implements the "tangent modulus" until the maximum load is reached.

EQMULT: Factor multiplying the equilibrium correction term. EQMULT = 0.0 indicates no equilibrium correction and EQMULT = 1.0 indicates full equilibrium correction.

III. Node Specification (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the

user can also utilize a shorthand form of the input. That is, specifying m and $-n$ consecutively is the equivalent of the specification of nodes $m, m + 1, m + 2, \dots, n$ and specifying $m, -p,$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m + p, m + 2p, \dots, m + kp$ where $m + kp$ is the highest integer of the form less than or equal to n . For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Member Connectivity (Node Number of Each Member)

FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type:

TRIM - Triangular membrane element (Ref. 3)

BEAM - Beam element

STRG - Stringer element

The first integer field designates the member number. The next integer fields designate the connecting nodes as follows:

TRIM - The nodes for the triangular family of elements are specified around the perimeter beginning with a major (vertex) node. Then a minor (midside) node and then alternatively major and minor as shown in Fig. 1. The absence of a minor node must be indicated by a zero or blank field in the proper position.

BEAM - Three node specifications are necessary for the beam element.

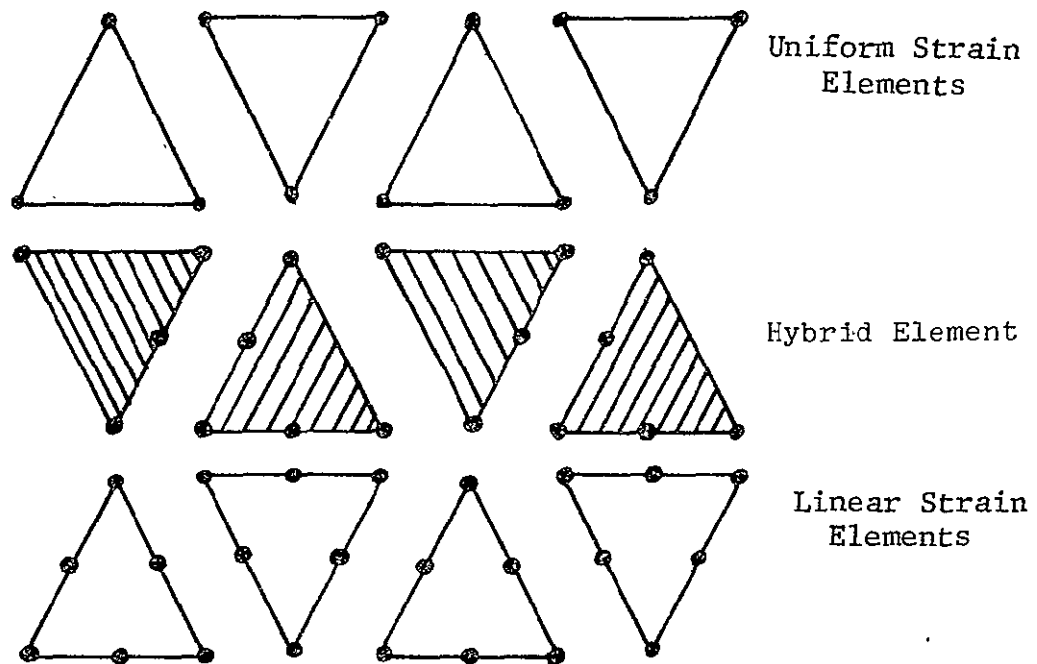


Fig. 1a Triangular Family of Finite Elements Used

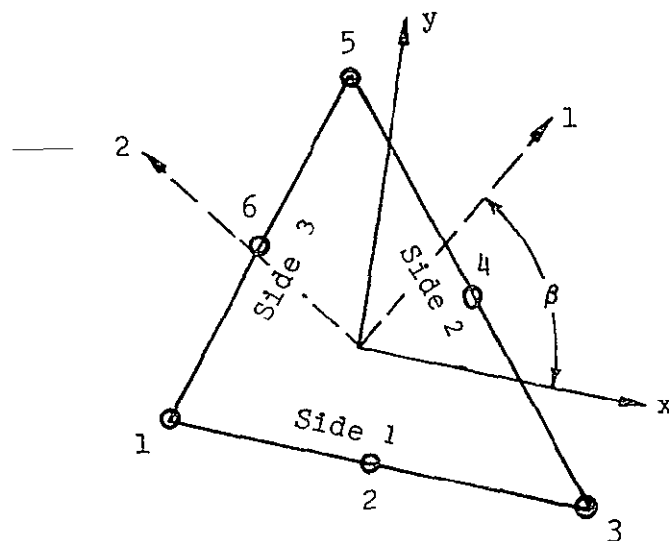


Fig. 1b Elements Topology

Nodes i and j (Fig. 2) which designate the element end points and a third node k , defining the normal to the beam axis about which the section properties are specified. This additional node may be a node of the structural idealization or it may be a "fictitious node" specified just for the purpose of defining the beam section properties. This is shown in Fig. 2.

STRG - Three node specifications are necessary for the stringer element. Nodes i and j , connecting the end points, and if desired an additional node designating a midpoint node. This is shown in Fig. 3. A zero or blank for the midpoint node specifies a two-node stringer. The midside node is the third node specified.

SEND Ends the section

V. X-Coordinates of Nodes FORMAT (E15.7, 13I5)

The x -coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5-field terminates the card scan for a given x -coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.

VIII. Boundary Conditions FORMAT (6I1, 9X, 13I5)

The first six fields give the boundary conditions specifications in the order: $u, v, w, \theta_x, \theta_y, \theta_z$; where u, v, w are global displacements in the x, y, z directions, respectively, and $\theta_x, \theta_y, \theta_z$ are rotations with respect to the x, y, z axes. The last three fields for $\theta_x, \theta_y, \theta_z$ are only used when using a beam element.

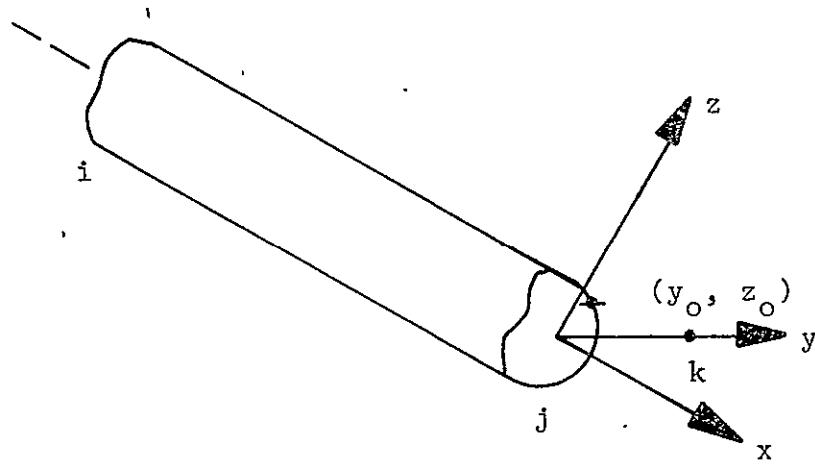


Fig. 2 Beam Element

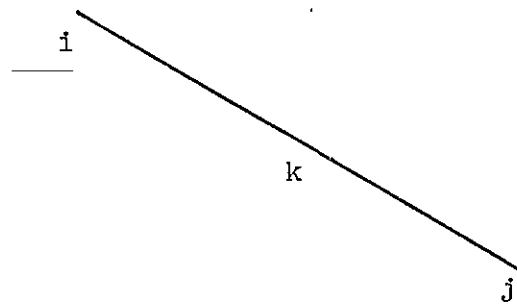


Fig. 3 Stringer Element

Zero denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 13I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 13I5 fields are used on a continuation card. A zero or blank I5 field terminates the card scan for a given boundary condition specification. Note: If the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default (i.e., in the absence of beams, 111000) with all nodes used in the problem specified. Note: Maximum number of nodes is currently 900.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacement of the form:

$$1) \quad \delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \dots + \alpha_n \delta_n$$

where δ_i is a dependent degree of freedom,

$\delta_{j1} \dots \delta_{jn}$ are independent degrees of freedom

and $\alpha_1, \alpha_2, \dots, \alpha_n$ are coefficients

2) Rotation of displacements at a node

$$\delta_i = \alpha_1 \delta_i + \alpha_2 \delta_j + \alpha_3 \delta_k$$

$$\delta_j = \beta_1 \delta_i + \beta_2 \delta_j + \beta_3 \delta_k$$

$$\delta_k = \gamma_1 \delta_i + \gamma_2 \delta_j + \gamma_3 \delta_k$$

where the α , β , γ 's are the direction cosines of the rotation, δ_i , δ_j , δ_k are the displacements with respect to the original global directions, and δ_i , δ_j , δ_k are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

$$\delta_i = \alpha_1$$

where the coefficient α_1 is the applied generalized displacement.

The first two fields designate a node number and a degree of freedom (i.e., 1-6). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material and Section Properties

The format for this input is dependent upon the member type. Each type of input begins with a word of up to four characters.

MAT1	Material properties for plane stress, necessary with membrane triangles (TRIM)
MBM	Material properties for a beam element
MSTG	Material and section properties for a stringer
THIK	Member thickness for triangular membrane (TRIM) elements
MBET	Angle between local axes and principal directions of orthotropy for TRIM elements
SREC	Beam section properties for a solid rectangular section
SCIR	Beam section properties for a solid circular section
ZSEC	Beam section properties for a Z-section
ISEC	Beam section properties for an I-section
HCIR	Beam section properties for a hollow circular section
HREC	Beam section properties for a hollow rectangular section
LSEC	Beam section properties for an L-section
TSEC	Beam section properties for a T-section
CSEC	Beam section properties for a channel section
SEND	Ends the section

MAT1 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /, 2E15.7, /, 4E15.7, /, 5E15.7, /, (16I5))

The first four cards specify material properties, as follows:

CARD 1: MAT1

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

BETA = No longer used. Set equal to zero.

GONTO = Shear modulus in (1)-(2) principal property plane

VONTO = Poisson's ratio, ν_{12}

$$\text{Note: } \epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2} \sigma_2$$

$$\epsilon_2 = -\frac{\nu_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2}$$

$$\nu_{12} = \frac{\tau_{12}}{G_{12}}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in
1-axis direction, α_1

TALF-2 = Coefficient of thermal expansion in
2-axis direction, α_2

CARD 3: SIGOX = Yield stress in principal 1-direction
SIGOY = Yield stress in principal 2-direction
SIGOZ = Yield stress in principal 3-direction
SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter
used in Ramberg-Osgood representation of
stress-strain behavior

If RMOSS = 0; RMOSN = $\bar{\alpha}$ the slope of the
linear strain hardening stress-strain repre-
sentation, i.e., $\bar{\alpha} = E_T/E$ where E_T is
the tangent modulus

RMOSS = If RMOSS $\neq$ 0; RMOSS = Ramberg-Osgood
parameter $\sigma_{0.7}$

Note 1: If RMOSN = 0 and RMOSS = 0,
the material for the element(s)
is assumed to be elastic-ideally
plastic

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

YLDST = Yield stress in tension

YLDSC = Yield stress in compression

Input for the yield stress in tension and compression has been maintained in order to accommodate materials that exhibit initial anisotropic plastic behavior. In this case an initial translation of the yield surface is made consistent with the kinematic hardening theory.[†]

Note: Only initially isotropic materials can be treated when considering linear or nonlinear strain hardening using this property card.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MBET - Orientation of Axes of Material Anisotropy - FORMAT (A4, 1X,
E15.7, /, (16I5))

CARD 1: MBET

BETF = Angle θ in degrees between local x-axis and
principal 1-axis for material orthotropy. See
Fig. 4. Only applicable for TRIM elements.

[†]Isakson, G., Armen, H. Jr., and Pifko, A., "Discrete-Element Methods for the Plastic Analysis of Structures," NASA Contractor's Report NASA CR-803, October 1967.

Note: This card is an optional card. The default is $BETF = 0.0$ for all members. It should be used with initially orthotropic materials.

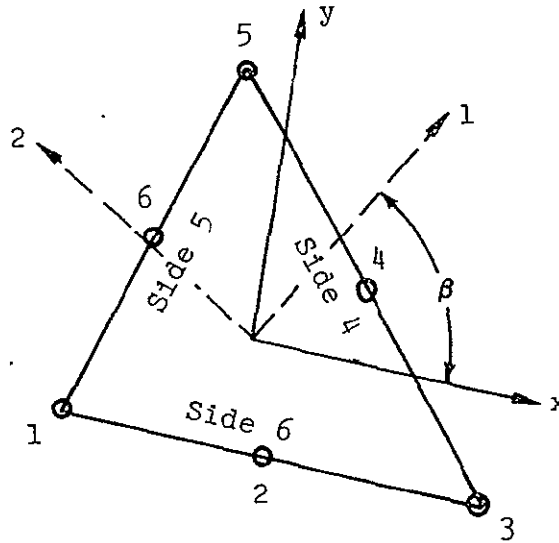


Fig. 4 Orientation of Element Local and Material Axes

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MSTG - Stringer Properties - FORMAT (A4, 1X, 5E15.7, /E15.7/, (16I5))

CARD 1: MSTG

E = Young's modulus

A = Cross sectional area

RMOSN = If $RMOS \neq 0$; $RMOSN = n$, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If $RMOSS = 0$; $RMOSEN = \bar{\alpha}$, the slope of the linear strain hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

$RMOSS =$ If $RMOSS \neq 0$; $RMOSS =$ Ramberg-Osgood parameter $\sigma_{0.7}$

Note 1: If $RMOSEN = 0$ and $RMOSS = 0$, the material for the element(s) is assumed to be elastic-ideally plastic

$YLDST =$ Yield stress

CARD 2: $TALF =$ Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

THIK - Element Thicknesses - FORMAT (A4, 1X, E15.7)

Necessary with membrane elements

TRIM

CARD 1: THIK

THICK = Element thickness

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MBM - Beam Material Properties - FORMAT (A4, 1X, 5E15.7, /, E15.7, /, (16I5))

CARD 1: MBM

E = Young's modulus

ANU = Poisson's ratio

RMOSS = If not equal to zero, RMOSS equals
Ramberg-Osgood parameter, $\sigma_{0.7}$

RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter
used in Ramberg-Osgood representation of stress-
strain behavior.

If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the
linear strain hardening stress-strain repre-
sentation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the
tangent modulus

YLDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Sec-
tion III are permitted. Any number of continuation cards may be
used for a given specification. A zero or blank I5 field ends
each member listing.

Beam Section Properties - FORMAT (A4, 1X, 5E15.7, /, 3E15.7, /,
formats for Cards 3 and 4 (see
below), /, (16I5))

The cards specifying beam section properties all start with
the following information:

CARD 1:	SREC	} Defined at the beginning of the section. The different cross sections are shown in Tables 1 and 2.
	SCIR	
	ISEC	
	ZSEC	
	HCIR	
	HREC	
	LSEC	
	TSEC	
	CSEC	

A = cross-sectional area
 I_{yy} = moment of inertia about y axis (see Fig. 5)
 I_{zz} = moment of inertia about z axis (see Fig. 5)
 I_{yz} = product of inertia
 J = torsional rigidity

CARD 2: Y_0 = Eccentricity of attachment point in the y_0 direction
 Z_0 = Eccentricity of attachment point in the z_0 direction
 $\bar{\beta}$ = angle defining the transformation of the I_y, I_z, I_{yz} to another reference axis (see Fig. 5)

Additional cards are required according to which section is specified. The notation for each section is shown in Table 1.

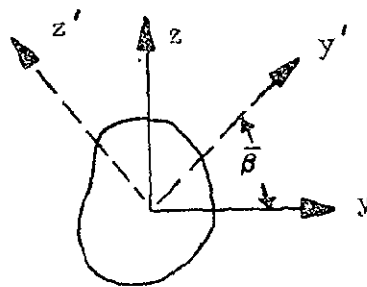


Fig. 5 Definition of Coordinate Axes in Cross Section of Beam Element

SREC - FORMAT (2E15.7)

CARD 3: A = Width
B = Depth

SCIR - FORMAT (E15.7)

CARD 3: R = Radius

ZSEC, ISEC, and CSEC - FORMAT (3E15.7, /, 3E15.7)

CARD 3: A1 = Dimension of upper flange
A2 = Dimension of web
A3 = Dimension of lower flange

CARD 4: T1 = Thickness of upper flange
T2 = Thickness of web
T3 = Thickness of lower flange

HCIR - FORMAT (2E15.7)

CARD 3: R = Outer radius
T = Thickness

HREC, LSEC, and TSEC - FORMAT (4E15.7)

CARD 3: A1 = Width (parallel to y-axis)
A2 = Depth (parallel to z-axis)
T1 = Thickness of upper and/or lower flanges
T2 = Thickness of vertical webs

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

SEND Ends the section.

TABLE 1 SECTION INPUT AND GEOMETRY

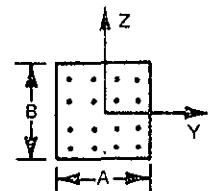
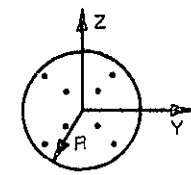
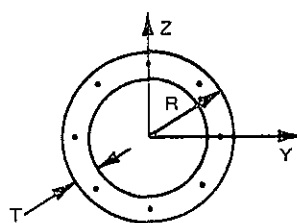
Section	Keyword	Input	
Solid Rectangle	SREC	Area, moments of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Section width and height, a, b.	
Solid Circular	SCIR	Area, moments of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Section radius, r.	
Hollow Circular Section	HCIR	Area, moment of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Outer radius, r, thickness, t.	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

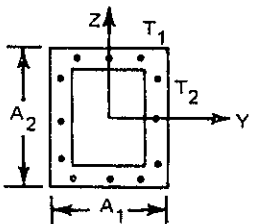
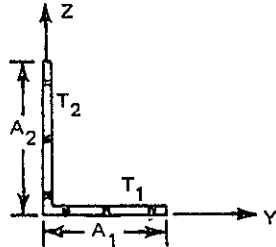
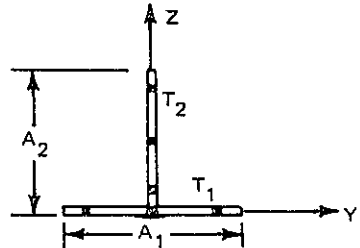
Section	Keyword	Input	
Hollow Rectangular Section	HREC	Area, moment of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Width, a_1 , depth, a_2 , thickness of upper and lower flanges, t_1 , thickness of vertical webs, t_2	
L-Section	LSEC	Area, moment of inertia, I_{yy} , I_{zz} , I_{yz} , J eccentricities, y_o , z_o , transformation angle $\bar{\beta}$. Dimension of flange and web a_1, a_2 thickness flange, and web, t_1 , t_2 .	
T-Section	TSEC	Same as for L-Section	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

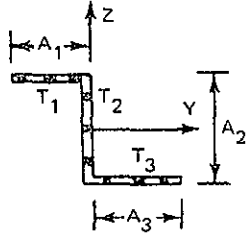
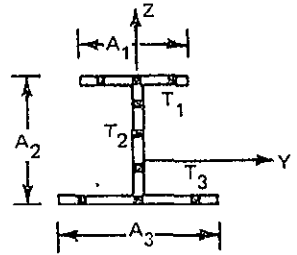
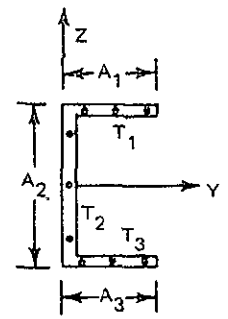
Section	Keyword	Input	
Z-Section	ZSEC	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Dimension of upper and lower flange, a_1, a_3. Dimension of web, a_2, thickness of upper and lower flange, t_1, t_3, thickness of web, t_2 NOTE: $a_1 = a_3$; $t_1 = t_3$	
I-Section	ISEC	Same as Z-section $a_1 \neq a_3$; $t_1 \neq t_3$	
Channel Section	CSEC	Same as Z-Section $a_1 = a_3$; $t_1 = t_3$	

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS

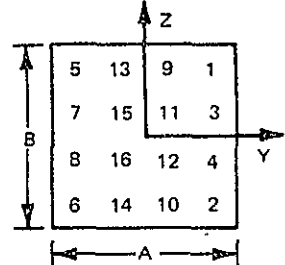
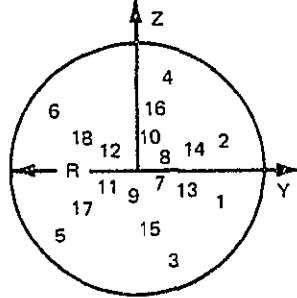
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
Solid Rectangle	SREC	16	$\pm y$	$\pm z$	 Shear Center at Centroid
			1	0.430568a	0.430568b
			3	0.430568a	0.169991b
			9	0.169991a	0.430568b
			11	0.169991a	0.169991b
Solid Circular	SCIR	18	r	$\pm \theta^\circ$	 Shear Center at Centroid
			2	0.887a	12.1
			4	0.887a	60.98
			6	0.887a	137.05
			8	0.1127a	12.1
			10	0.1127a	60.98
			12	0.1127a	137.05
			14	0.500a	12.1
			16	0.500a	60.98
			18	0.500a	137.05

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

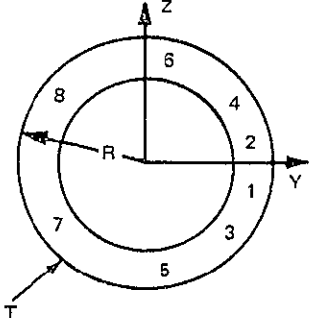
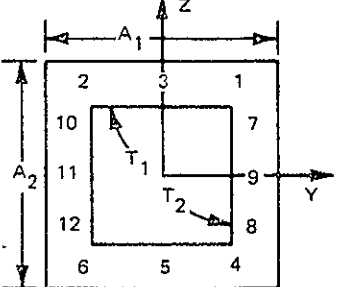
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
Hollow Circular Section	HCIR	8	r	θ°	 Shear Center at Centroid
			1	a	352.8
			2	a	7.2
			3	a	323.4
			4	a	36.6
			5	a	274.6
			6	a	85.4
			7	a	213.0
			8	a	147.0
Hollow Rectangular Section	HREC	12	$\pm y$	$\pm z$	 Shear Center at Centroid
			1	$0.77459\bar{a}_1$	$0.5\bar{a}_2$
			3	0	$0.5\bar{a}_2$
			7	$0.5\bar{a}_1$	$0.77459\bar{a}_2$
			9	$0.5\bar{a}_1$	0
				$\bar{a}_1 = a_1 - t_2$	
				$\bar{a}_2 = a_2 - t_1$	

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

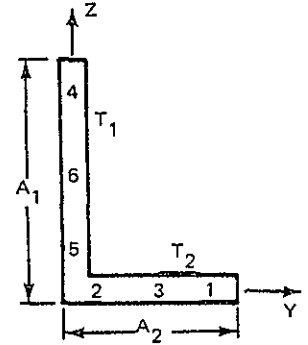
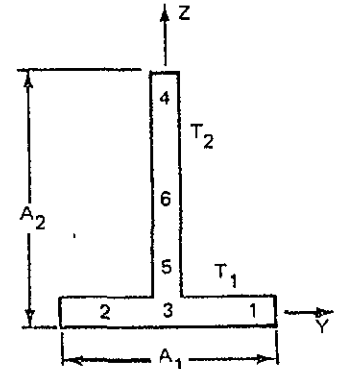
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
L-Section	LSEC	6	y	z	 Axes at Shear Center
			1	$0.88730\bar{a}_2$	0.0
			2	$0.11271\bar{a}_2$	0.0
			3	$0.50000\bar{a}_2$	0.0
			4	0.0	$0.88730\bar{a}_1$
			5	0.0	$0.11271\bar{a}_1$
			6	0.0	$0.50000\bar{a}_1$
			$\bar{a}_1 = a_1 - \frac{t_2}{2}, \quad \bar{a}_2 = a_2 - \frac{t_1}{2}$		
T-Section	TSEC	6	y	z	 Axes at Shear Center
			1	$0.77459a_1$	0.0
			2	$-0.77459a_1$	0.0
			3	0.0	0.0
			4	0.0	$0.88730\bar{a}_2 + \frac{t_1}{2}$
			5	0.0	$0.11271\bar{a}_2 + \frac{t_1}{2}$
			6	0.0	$0.50000\bar{a}_2 + \frac{t_1}{2}$
			$\bar{a}_2 = a_2 - t_1$		

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

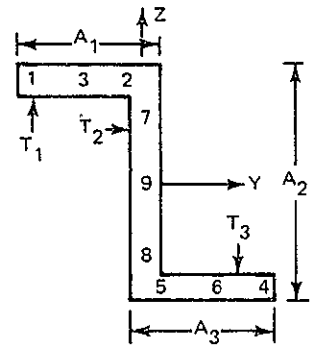
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
Z-Section	ZSEC	9	y	z	 Shear Center at Centroid
			1 $-0.88730a_1$	$\bar{a}_2$	
			2 $-0.11271a_1$	$\bar{a}_2$	
			3 $-0.5000a_1$	$\bar{a}_2$	
			4 $0.88730a_1$	$\bar{a}_2$	
			5 $0.11271a_1$	$\bar{a}_2$	
			6 $0.5000a_1$	$\bar{a}_2$	
			7 0.0	$0.77459a_2'$	
			8 0.0	$0.77459a_2'$	
			9 0.0	0.0	
			$\bar{a}_2 = (a_2 - t_1)/2$, $t_1 = t_3$		
			$a_2' = (a_2 - t_1)$, $a_1 = a_3$		

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

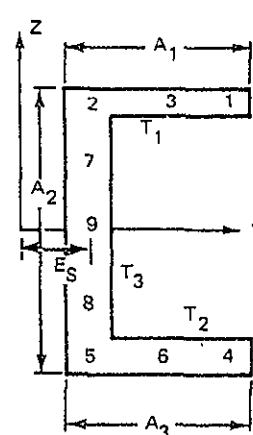
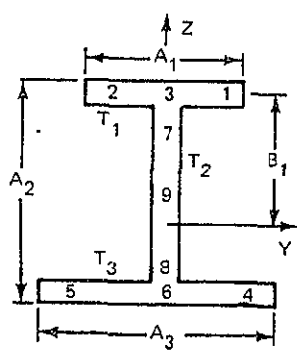
Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points	
Channel Section	CSEC	9	y	z	 Axes at Shear Center	
			1	$0.88730\bar{a}_1 + e_s$		$0.5\bar{a}_2$
			2	$0.11271\bar{a}_1 + e_s$		$0.5\bar{a}_2$
			3	$0.5000\bar{a}_1 + e_s$		$0.5\bar{a}_2$
			4	$0.88730\bar{a}_1 + e_s$		$-0.5\bar{a}_2$
			5	$0.11271\bar{a}_1 + e_s$		$-0.5\bar{a}_2$
			6	$0.5000\bar{a}_1 + e_s$		$-0.5\bar{a}_2$
			7	e_s		$0.77459\bar{a}_2$
			8	e_s		$-0.77459\bar{a}_2$
			9	e_s		0.0
			e_s - distance from shear center to web center line $e_s = \frac{1/2(a_1 - t_2)}{1 + 1/6 \left(\frac{t_2(a_2 - t_1)}{t_1(a_1 - t_2)} \right)}, \quad \bar{a}_2 = a_2 - t_1$			
			NOTE: $t_1 = t_3$; $a_1 = a_3$			

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

Section	Keyword	Number Stress Points	Location of Stress Points		Location of Stress Points
I-Section	ISEC	9	y	z	 Axes at Shear Center
			1	$0.774569a_1$	b_1
			2	$-0.77459a_1$	b_1
			3	0.0	b_1
			4	$0.77459a_3$	b_2
			5	$0.77459a_3$	b_2
			6	0.0	b_2
			7	0.0	$0.8873\bar{b}_1 + 0.11271\bar{b}_2$
			8	0.0	$0.11271\bar{b}_1 + 0.8873\bar{b}_2$
			9	0.0	$0.5\bar{b}_1 + 0.5\bar{b}_2$
			$\bar{a}_2 = \frac{\bar{a}_2 \left(\frac{t_3}{t_1} \right) \left(\frac{a_3}{a_1} \right)^3}{\left(1 + \left(\frac{t_3}{t_1} \right) \left(\frac{a_3}{a_1} \right) \right)^3}, \quad b_2 = b_1 \left(\frac{t_1}{t_3} \right) \left(\frac{a_1}{a_3} \right)^3$		
			$\bar{a}_2 = a_2 - 0.5(t_1 + t_3); \quad \bar{b}_1 = b_1 - 0.5t_1; \quad \bar{b}_2 = b_2 - 0.5t_3$		

XI. Applied Loads

Two different types of loading are currently admissible — concentrated loads at nodes, and distributed line loads on beams. Each loading situation is designated by one of the following four-character key words:

CONC	(Concentrated force or moment at a node)
BMLO	(Distributed line load, beam element)
SEND	(Section end)

Note: The keywords for the section on applied loads are used as section headings. The keyword appearing on an input card designates that the input to follow is associated with a particular type of applied loading. A blank card (as described in succeeding sections) delimits the input and indicates that the next card contains a different keyword. For example, CONC indicates input that follows is for concentrated loads and TRIA; distributed loads on an edge of a membrane triangle. Thus the input is as follows:

```
CONC
      data
      for
      concentrated
      loads
      blank card
BMLO
SEND
```

It should be noted that the keywords can appear in any order and may be specified more than one time in the applied load section.

CONC - Concentrated loads - FORMAT (A4, /, I5, 3E15.7, /, 5X, 3E15.7)

Each card gives the load components in global directions at a specified node.

CARD 1: CONC

CARD 2:	I5 field:	Node number
	First E15.7 field:	Force component F_x
	Second E15.7 field:	Force component F_y
	Third E15.7 field:	Force component F_z
CARD 3:	First E15.7 field	Moment component M_x
	Second E15.7 field	Moment component M_y
	Third E15.7 field	Moment component M_z

A blank card (i.e., zero or blank first I5 field) ends the specification of concentrated loads.

BMLO - Distributed Line Load, Beam Element - FORMAT (A4, /, 4E15.7)

CARD 1: BMLO

CARD 2:	PYI	Force/unit length in local y-direction (see Fig. 5) at node i
	PYJ	Force/unit length in local y-direction at node j
	PZI	Force/unit length in local z-direction at node i
	PZJ	Force/unit length in local z-direction at node j

Note: A linear variation of the distributed load between nodes is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

A card with SEND in the first four columns ends the section for applied loading.

XII. Members to be Printed — FORMAT (16I5)

Specify the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A maximum of 600 members may be specified. Members in excess of 600 and undefined member numbers are ignored. A blank card or card with only zero entries ends the section.

XIII. Nodes to Be Printed — FORMAT (16I5)

Up to 900 nodes whose displacements are to be printed for the analysis, as per Section XII.

XIV. Parameters for Succeeding Load Cycles — FORMAT (3I5, /, 3E15.7)

CARD 1: NPRNT = If equal to zero, no additional load cycle, end of problem. If greater than zero, print output every NPRNT increments of load
 NFORM = Reform elastic stiffness matrix every NFORM increments of load
 NNEWL = If equal to zero, no new load distribution is to be input. If greater than zero, a new load distribution will be read in

CARD 2: PMAX = New maximum load factor
 DELP = Number of load increments for new load cycle
 PTAN = Load at which full tangent modulus is used

XV. New Applied Loads

New load distribution is read in if $NNWL \neq 0$. Format is identical to Section XI.

XVI. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (2A4)

As in Section I

CARD 2: NPNTC, NPRNT, IFORM, IREST, NUTAP, INPRT, NRSRT

As in Section II. Here NPNTC is ignored and
IREST = 2

NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form:

* * * * *

RESTART TAPE HAS BEEN WRITTEN FOR P = 7.100000E+04
NRSRT = 8

* * * * *

In this case NRSRT is equal to 8 and the job will be restarted at a load factor of 71.0. A restart job can be started at any intermediate load level for which the restart tape has been written.

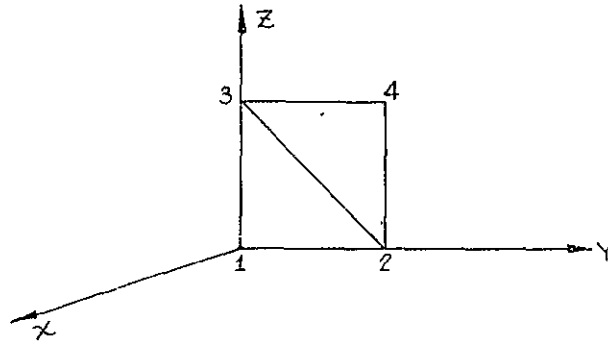
CARD 3: PMAX, DELP, PTAN, EQMULT

As in Section II

Succeeding cards follow as in Sections XII through XVI.

APPENDIX I -- MULTIPOINT CONSTRAINTS

EXAMPLE 1



Node 1 and 2 are fixed.

There is an applied displacement at node 3 in the negative y-direction.
The distance between nodes 3 and 4 remain constant.

Boundary Conditions

011000	1-4	default condition
000000	1 2 4	
021000	3	
Blank card		

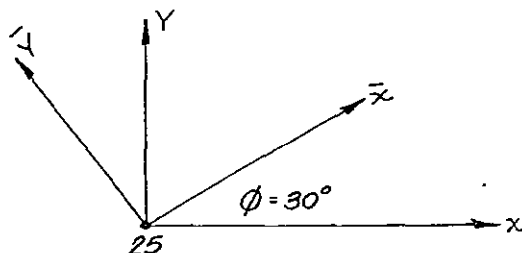
Dependencies

Node	Dof	Node	Dof		
3	2	3	2	0.050	(specified displacement)
4	2	3	2	1.0	restraint condition that
4	3	3	3	1.0	3-4 remain rigid

NOTE: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).

EXAMPLE 2

Rotation of Displacements at a Boundary



For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are x, y . Thus:

$$u = \bar{u} \cos \phi - \bar{v} \sin \phi$$

$$v = \bar{u} \sin \phi + \bar{v} \cos \phi$$

The multipoint constraint cards are:

25	1	25	1	0.86602540E00	25	2	-0.50E00
25	2	25	1	0.50E00	25	2	0.86602540E00

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.

INSTRUCTIONS FOR USE
OF SATELLITE

A Preprocessing Program
for PLANS

INSTRUCTIONS FOR USE OF SATELLITE

SATELLITE is the preprocessing program associated with the PLANS system programs that checks and plots the undeformed finite element model. Additionally, it calculates the ordering of the external node numbers that leads to the minimum semibandwidth. SATELLITE is currently not operational for the HEX program.

The input for SATELLITE is as follows:

CARD 1: FORMAT (20A4)

Keywords left justified in fields of four. One of the following is required and indicates the program whose data follows.

PLNE	- PLANE program, In-Plane Membrane Program Used for Fracture Analysis
BEND	- BEND program, Elastic-Plastic Analysis of Built-Up Structures
REVB	- REVBY program, Elastic-Plastic Analysis of —— Bodies of Revolution
OPGM	- OUT-OF-PLANE GM, Nonlinear Analysis of Built-Up Structures
DYCA	- DYCAST program, Nonlinear Dynamic Analysis of Built-Up Structures

Additional optional keywords can be specified in fields of four. Any number of spaces can be skipped on the input card that are multiples of four.

BAND	- Specifies that the optimum semibandwidth is to be calculated. In this case the optimum order of the external node numbers for
------	---

Section III data will be printed and punched. These punched data are used with the input deck. If this keyword is not specified the bandwidth will not be optimized.

SCAN - Process and check input data but do not produce any plots

PRIN - Print transformed coordinates to be plotted

The input data deck follows Card 1. Input through the nodal coordinates is necessary. The remainder of the deck may be input although the cards are read and not processed until a STOP or END card is reached. Input for the SATELLITE program continues as follows.

CARD 2: FORMAT (5I5)

This card determines the number and type of plots. Up to five different labeled pictures can be obtained. These are specified with the fixed point numbers right justified in any of the fields of five as follows:

- 1 - labeled unconnected nodes
- 2 - labeled nodes, unlabeled members
- 3 - labeled nodes and members
- 4 - unlabeled nodes, labeled members
- 5 - unlabeled nodes and members

CARD 3: FORMAT (4E15.7)

ALPHA - rotation of structure about the global x-axis

BETA - rotation of structure about the global y-axis

GAMMA - rotation of structure about the global z-axis ..
LENGTH - maximum width of picture in the paper transverse direction. LENGTH is a function of the plotter being used.

If ALPHA, BETA, GAMMA are zero then the picture is projected on the x-y plane with the y direction in the transverse paper direction. The angles lead to a path dependent transformation by rotating the body first about the global x then y and z axes, respectively. The resulting viewing plane is still the original global x-y plane. The viewing plane can be changed with the following card.

CARD 4: FORMAT (6A1)

This card specifies the viewing plane to be used by specifying the six alphanumeric characters XYZ+++. The first two characters specifying the viewing plane with the first indicating the length direction and the second the width of the paper. The last three characters can be the characters + or - and indicate that the normal to the plane is in the positive or negative coordinate direction. For example:

YZX+++ indicates the viewing plane as the global YZ plane with the Z axis in the paper width direction and Y along the paper length

CARD 5: FORMAT (4(A4,1X))

Element types to be omitted. For BEND, OPGM, DYCAST:

BEAM - will omit all beam elements

TRIM - will omit all triangle membrane elements

STGR - will omit all stringer elements
TRIP - will omit all triangular plate elements

For REVBY:

TRIR - will omit all triangle elements
SHEL - will omit all shell elements
RING - will omit all ring elements

If there are no element types to be omitted a blank card must be specified.

CARD 6: FORMAT (16I5)

Nodes to be included in the plot. All the shorthand notations are allowed, i.e., 1 through 100 is input as 1 - 100 and 1, 3, ..., 99 as 1-2-99. A blank card or card with only zero entries ends the section.

If only a part of the structure is to be plotted, this section should include only the nodes that lie in the section. Only these nodes are considered when scaling the plot so that the subsection will be scaled up to the maximum allowable size.

CARD 7: FORMAT (16I5)

Elements to be included in the plot. Input is the same as for Card 6. All nodes associated with the elements to be plotted must be specified with Card 6.

Note: If all the elements and nodes are to be plotted it is sufficient to specify one negative number in the first I5 field whose magnitude is greater than the largest external element/node number in the finite element grid.

CARD 8: FORMAT (A4)

 If this card is blank and

- 1 END was read at the end of the data deck a new
 data deck is read starting from Card 1
- 2 STOP was read at the end of the data deck, the
 . job is complete

If this card is REPT the input will be repeated from Card 2
and the same structure with different plotting parameters will
be plotted.

SECTION 7

EXAMPLE INPUT

EXAMPLE INPUT

The following pages contain the input data decks for a number of sample problems. The principal intent in providing sample input is to aid the user in better understanding the discussion in the main body of the users' manual. Also provided with each problem is a brief discussion and representative data so that benchmark results are available for comparison. A detailed picture of the sample problem idealizations showing node and element labeling has not been provided. It is suggested that the input plotting program SATELLITE be used to generate computer plots of the input data, if desired.

Many sample problems that demonstrate the capability of the methods used in the PLANS program can be found in Refs. 1-3.

7-2

THE RESEARCH DEPARTMENT
OF GRUMMAN AEROSPACE CORPORATION
UNDER PARTIAL FUNDING FROM
THE NASA LANGLEY RESEARCH CENTER

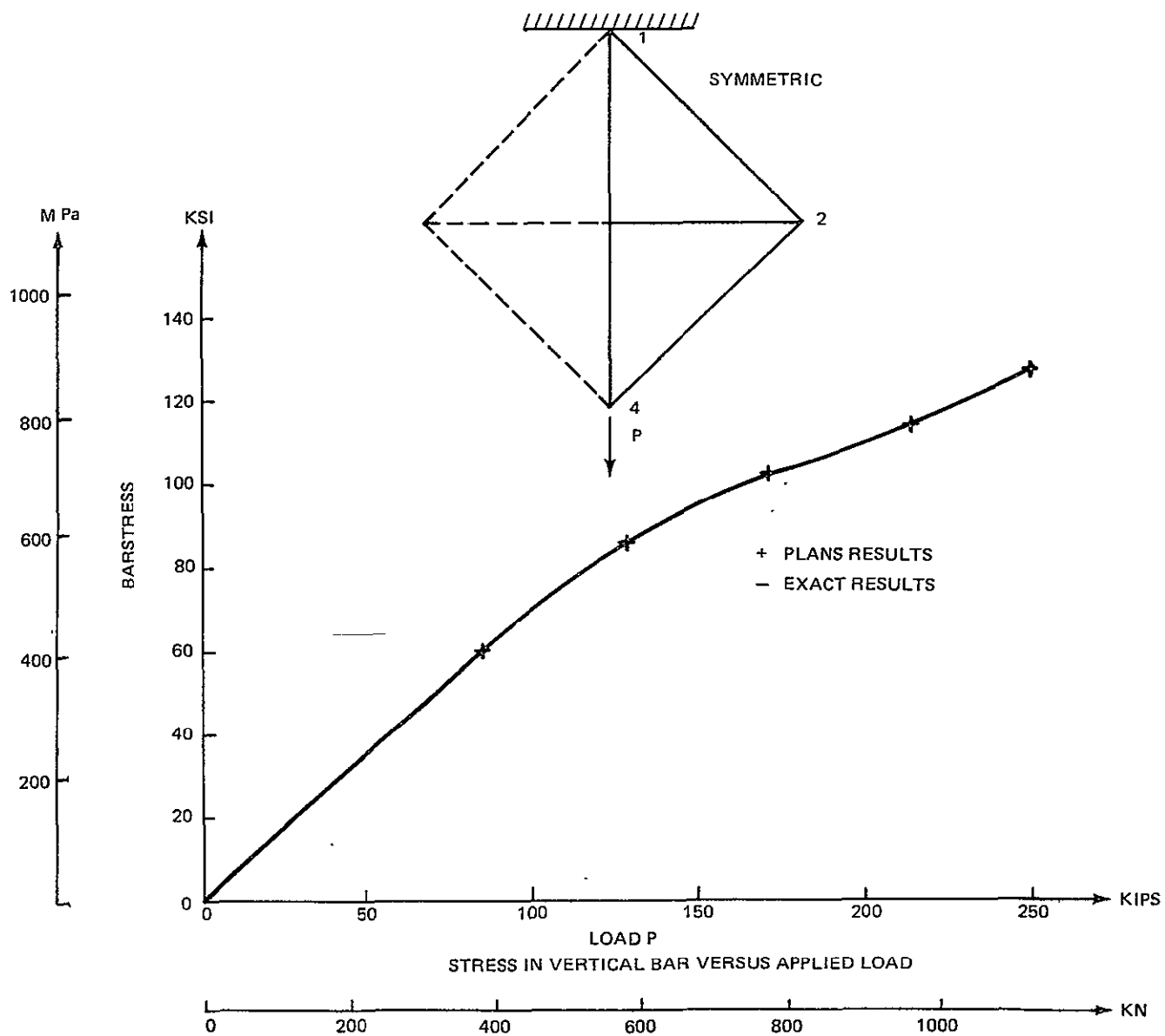
Example Problem No. 1

Program: BEND

Problem Title: Classical Truss - Axial Force Element

Comments: This problem uses the stringer element STGR in a simple 4 element truss. Symmetry conditions were used so that only half the truss was modeled.

The accompanying figure shows the applied load versus the force in the vertical element (element 3). These results were generated using a load step of $\Delta P = 848.5$ lbs or 3774.3 N (which is 0.01 of the initial yield load). Agreement with results from an exact analysis was quite good.



Bend Sample Problem # 1 Classical Truss STRG Axial Force Elements

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM #1 CLASSICAL TRUSS STRG AXIAL FORCE ELEMENTS

0 50
1 -4
STRG 1 1 2
STRG 2 2 4
STRG 3 2 3
STRG 4 1 4
SEND

0.0 1
.5 2 3
1.0 4

BLANK CARD

0.0 1 4 3
.5 2

BLANK CARD

0.0 1 -4

BLANK CARD

000000 1 3
110000 2
100000 4

BLANK CARD

BLANK CARD

MSTG 1.0 E6 07 1.0 E6 00 10.0 E6 00 1.0 E6 05 6.0 E6 04

BLANK CARD

1 -3
MSTG 1.0 E6 07 0.5 E6 00 10.0 E6 00 1.0 E6 05 6.0 E6 04

BLANK CARD

4
SEND
CONC 4 0.5 E6 00 0.0 E6 00 0.0 E6 00

BLANK CARD

BLANK CARD

SEND 1 -4

BLANK CARD

-4

BLANK CARD

250000.0 .01

BLANK CARD

STOP

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

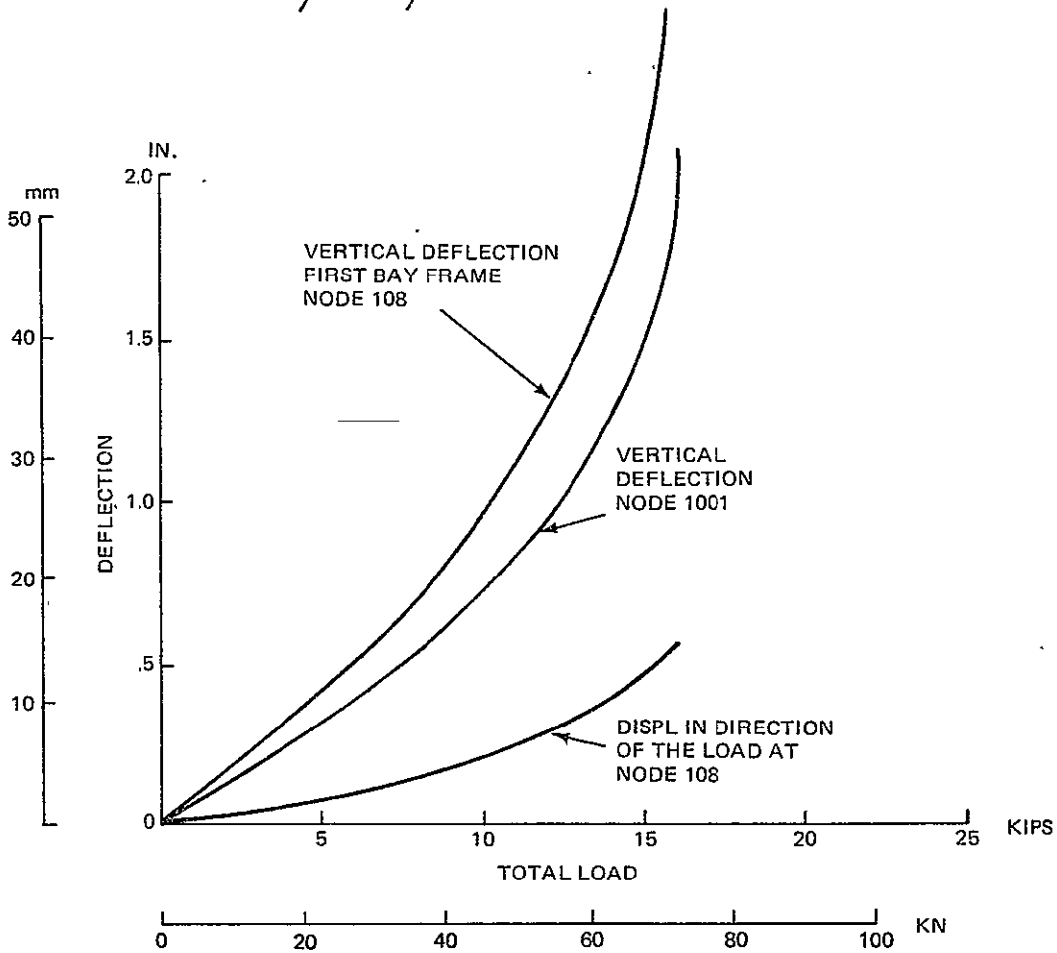
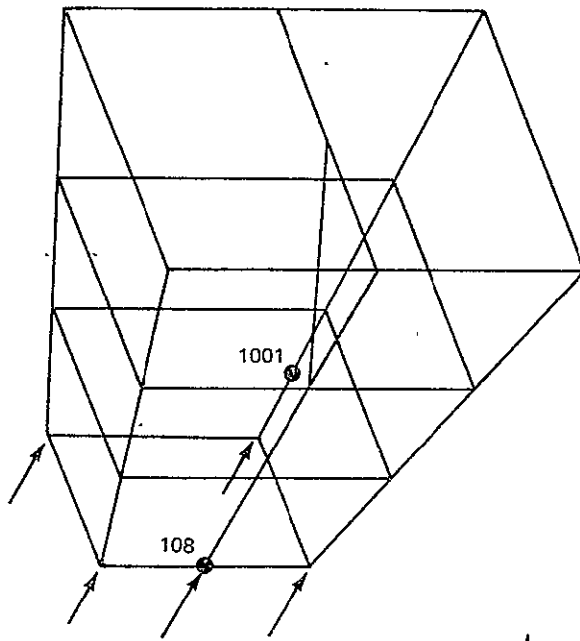
Example Problem No. 2

Program: BEND

Problem Title: NASA Frame with an End Load - BEAM ELEMENTS

Comments: This problem demonstrates the use of beam elements in an open space framework. Fifty seven beam elements of various cross section (L-section, T-section, rectangular section) connecting 40 nodes are used in the model. Multipoint constraints were used to enforce symmetry conditions between the left and right hand side of the structure. Ideally plastic behavior was assumed with a yield stress of 50130 psi or 345.6 MPa.

The figure shows displacement in the vertical and axial directions at the first bulkhead at the centerline (node 108) and at the midpoint between the first two bulkheads in the upper stringer (node 1001). The results indicate a plastic collapse at a load of $P = 15000$ lbs or 66.7 KN.



Bend Sample Problem # 2 NASA Frame with End Load -

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM =2 NASA FRAME WITH AN END LOAD - BEAM ELEMENTS

	1	20															
	101	1001	102	105	201	2000	108	3001	1003	106	202	6000	205	303	4000	2002	
	208	3003	1005	206	304	6002	311	403	4002	2004	316	3005	312	408	6004	8000	
	411	400	4004	414	8002	422	8004	406									
BEAM			1	101	102	108											
BEAM			3	106	102	108											
BEAM			4	108	106	102											
BEAM			5	201	202	208											
BEAM			7	206	202	208											
BEAM			8	208	206	202											
BEAM			10	303	304	316											
BEAM			14	312	304	316											
BEAM			16	316	312	304											
BEAM			18	400	408	406											
BEAM			22	422	408	406											
BEAM			24	414	422	406											
BEAM			25	414	406	422											
BEAM			27	406	400	422											
BEAM			29	102	2000	108											
BEAM			30	2000	202	106											
BEAM			31	202	2002	106											
BEAM			32	2002	304	106											
BEAM			33	304	2004	106											
BEAM			34	2004	408	106											
BEAM			35	4000	106	102											
BEAM			36	206	4000	102											
BEAM			37	4002	206	102											
BEAM			38	312	4002	102											
BEAM			39	4004	312	102											
BEAM			40	422	4004	102											
BEAM			41	6000	108	311											
BEAM			42	208	6000	311											
BEAM			43	6002	208	311											
BEAM			44	316	6002	311											
BEAM			45	6004	316	311											
BEAM			46	414	6004	311											
BEAM			47	8000	316	311											
BEAM			48	8002	8000	311											
BEAM			49	8004	8002	311											
BEAM			50	406	8004	311											
BEAM			53	105	101	108											
BEAM			54	105	108	101											
BEAM			57	205	201	208											

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

```

BEAM      58  205  208  201
BEAM      64  311  303  316
BEAM      66  311  316  303
BEAM      68  403  400  406
BEAM      72  411  403  406
BEAM      73  411  414  406
BEAM      75 1001  101  105
BEAM      76  201 1001  105
BEAM      77 1003  201  105
BEAM      78  303 1003  105
BEAM      79 1005  303  105
BEAM      80  403 1005  105
BEAM      81  105 3001  101
BEAM      82 3001  205  101
BEAM      83  205 3003  101
BEAM      84 3003  311  101
BEAM      85  311 3005  101
BEAM      86 3005  411  101
SEND

      56.875E00  101  102  105  106  103
      39.875E00  201  202  205  206  203
      22.625E00  303  304  311  312  316
      C.C      E+00  403  406  408  411  414  422  400
      48.375E00 2000 4000 1001 3001 6000
      31.25E00  2002 4002 1003 3003 6002
      11.3125E00 2004 4004 1005 3005 6004 8002
      16.96875E00 8000
      5.65625E00 8004

      C.C      E+00  108  208  316  400  405  414 6000
      C.C      C+00 6004 8000 8002 3004 6002

      12.328125E00  102  106
      -12.328125E00  101  105
      16.031250E00  202  206
      -16.031250E00  201  205
      19.7890625E00  304  312
      -19.7890625E00  303  311
      -24.71875E00  403  411
      24.71875E00  408  422
      14.1796875E00 2000 4000
      17.91015625E00 2002 4002
      22.28390625E00 2004 4004
      -14.1796875E00 1001 3001
      -17.91015625E00 1003 3003
    
```

BLANK CARD

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

BLANK CARD

-22.25390625E00 1005 3005
C.C E+00 406
10.1015625E00 101 102
-10.1015625E00 105 106 108
13.1953125E00 201 202
-13.1953125E00 205 206 208
16.3203125E00 303 304
-16.3203125E00 311 312 316
-8.16015625E00 8002
20.4375E00 400 403 408
-20.4375E00 411 414 422
11.6484375E00 2000 1001
14.7578125E00 2002 1003
18.37890625E00 2004 1005
-11.6484375E00 4000 3001 6000
-14.7578125E00 4002 3003 6002
-18.37890625E00 4004 3005 6004
-12.24023438E00 8000
-4.0800078125E00 8004

BLANK CARD

101010 101 201 303
101010 6000 6002 6004 8000 8002 8004 108 208 316
101010 105 205 311
000000 400 403 408 406 411 414 422
000000 102 106 202 206 304 312
000000 1001 1003 1005 3001 3003 3005

BLANK CARD

102 1 101 1 1.0
106 1 105 1 1.0
102 3 101 3 1.0
106 3 105 3 1.0
102 5 101 5 1.0
106 5 105 5 1.0
202 1 201 1 1.0
206 1 205 1 1.0
202 3 201 3 1.0
206 3 205 3 1.0
202 5 201 5 1.0
206 5 205 5 1.0
304 1 303 1 1.0
312 1 311 1 1.0
304 3 303 3 1.0
312 3 311 3 1.0

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

7-11

304	5	303	5	1.0
312	5	311	5	1.0
1001	1	2000	1	1.0
1003	1	2002	1	1.0
1005	1	2004	1	1.0
3001	1	4000	1	1.0
3003	1	4002	1	1.0
3005	1	4004	1	1.0
1001	2	2000	2	-1.0
1003	2	2002	2	-1.0
1005	2	2004	2	-1.0
3001	2	4000	2	-1.0
3003	2	4002	2	-1.0
3005	2	4004	2	-1.0
1001	3	2000	3	1.0
1003	3	2002	3	1.0
1005	3	2004	3	1.0
3001	3	4000	3	1.0
3003	3	4002	3	1.0
3005	3	4004	3	1.0
1001	4	2000	4	-1.0
1003	4	2002	4	-1.0
1005	4	2004	4	-1.0
3001	4	4000	4	-1.0
3003	4	4002	4	-1.0
3005	4	4004	4	-1.0
1001	5	2000	5	1.0
1003	5	2002	5	1.0
1005	5	2004	5	1.0
3001	5	4000	5	1.0
3003	5	4002	5	1.0
3005	5	4004	5	1.0
1001	6	2000	6	-1.0
1003	6	2002	6	-1.0
1005	6	2004	6	-1.0
3001	6	4000	6	-1.0
3003	6	4002	6	-1.0
3005	6	4004	6	-1.0

BLANK CARD

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

0.0E00 0.0E00
 2.0E00 1.0E00
 1 3 4 5 7 8 10 14 16 18 22 24 68 72 73 53
 54 57 58 64 66 0.5E00 18.0E00 0.0E00 1.791
 SREC 6.0E00 0.0E00 0.0E00 1.0E00

25 27
 TSEC .4474E00 .0448331 .0729367 0.0 7.0293E-03
 0.0 .90625
 1.0 2.09 .2775 .09375

41 46
 TSEC .4474E00 .0448331 .0729367 0.0 7.0293E-03
 1.0 2.09 .2775 .09375

47 50
 LSEC .1211E00 .01099 .01099 -.007087 1.57675E-04
 .9375 .9375
 1.0 1.0 .0625 .0625

29 40 75 86
 SEND
 CONC

BLANK CARD

101 -.13

BLANK CARD

102 -.13

BLANK CARD

105 -.13

BLANK CARD

106 -.13

BLANK CARD

108 -.48

BLANK CARD

SEND
 29 -50

BLANK CARD

-9000

BLANK CARD

50000.0 .01

BLANK CARD

STOP

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

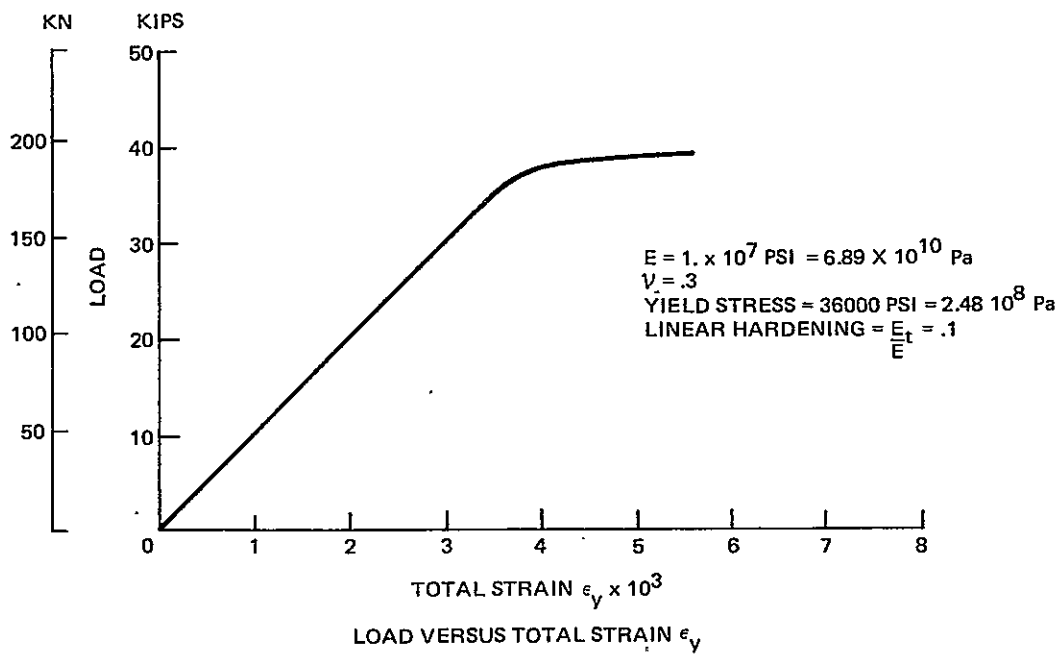
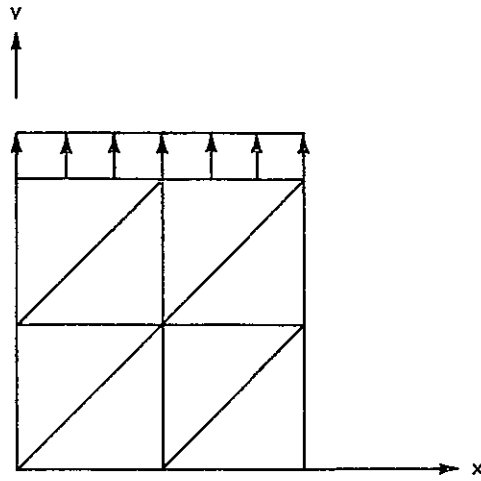
Example Problem No. 3

Program: BEND

Problem Title: 8 Element Square Plate

Comments: This problem represents a simple statically determinate case that uses all combinations of the triangular membrane element, TRIM. These range from 3-node constant strain triangles, the 6-node linear strain triangle, and the two interface cases with 4 and 5 nodes. A distributed edge load (using the TRIA feature) is applied on one edge.

The figure shows load versus longitudinal strain for a linear strain hardening material.



BEND SAMPLE PROBLEM #3 EIGHT ELEMENT SQUARE MEMBRANE

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM = 3 8 ELEMENT SQUARE PLATE TRIM ELEMENTS

101 -105- 202 -205 301 303 305 404 405 501 503 505
 TRIM 301 301 0 303 0 503 0
 TRIM 401 503 0 501 0 301 0
 TRIM 201 303 0 301 0 101 202
 TRIM 402 505 0 503 0 303 404
 TRIM 202 305 0 303 203 103 204
 TRIM 302 303 0 305 405 505 404
 TRIM 101 101 102 103 203 303 202
 TRIM 102 103 104 105 205 305 204

SEND
 0.0 E 00 101 301 501
 5.0 E 00 103 303 503
 10.0 E 00 105 305 505

BLANK CARD

0.0 E 00 101 103 105
 5.0000E 00 301 303 305
 10.0000E 00 501 503 505

BLANK CARD

0.0 E 00 101 103 105
 5.0000E 00 301 303 305
 10.0000E 00 501 503 505

BLANK CARD

000000 101
 100000 102 -105
 110000 202 -205 303 305 404 405 503 505
 010000 301 501

BLANK CARD

BLANK CARD

MAT1 1.0 E 07 1.0 E 07 0.0 2.0785 E 06 .3
 1.0 E-06 1.0 E-06
 3.6 E 04 3.6 E 04 3.6 E 04 2.0785 E 04
 .1 0.0 1.0 E 07 3.6 E 04 3.6 E 04
 101 102 201 202 301 302 401 402
 THIK 1.0
 101 102 201 202 301 302 401 402
 SEND
 TRIA
 501 503 301 0.0 -1.0
 503 505 303 0.0 -1.0
 0.0 -1.0

BLANK CARD

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

BLANK CARD
SEND
-1000

-1000

BLANK CARD
50000.0 .01

BLANK CARD
STOP

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

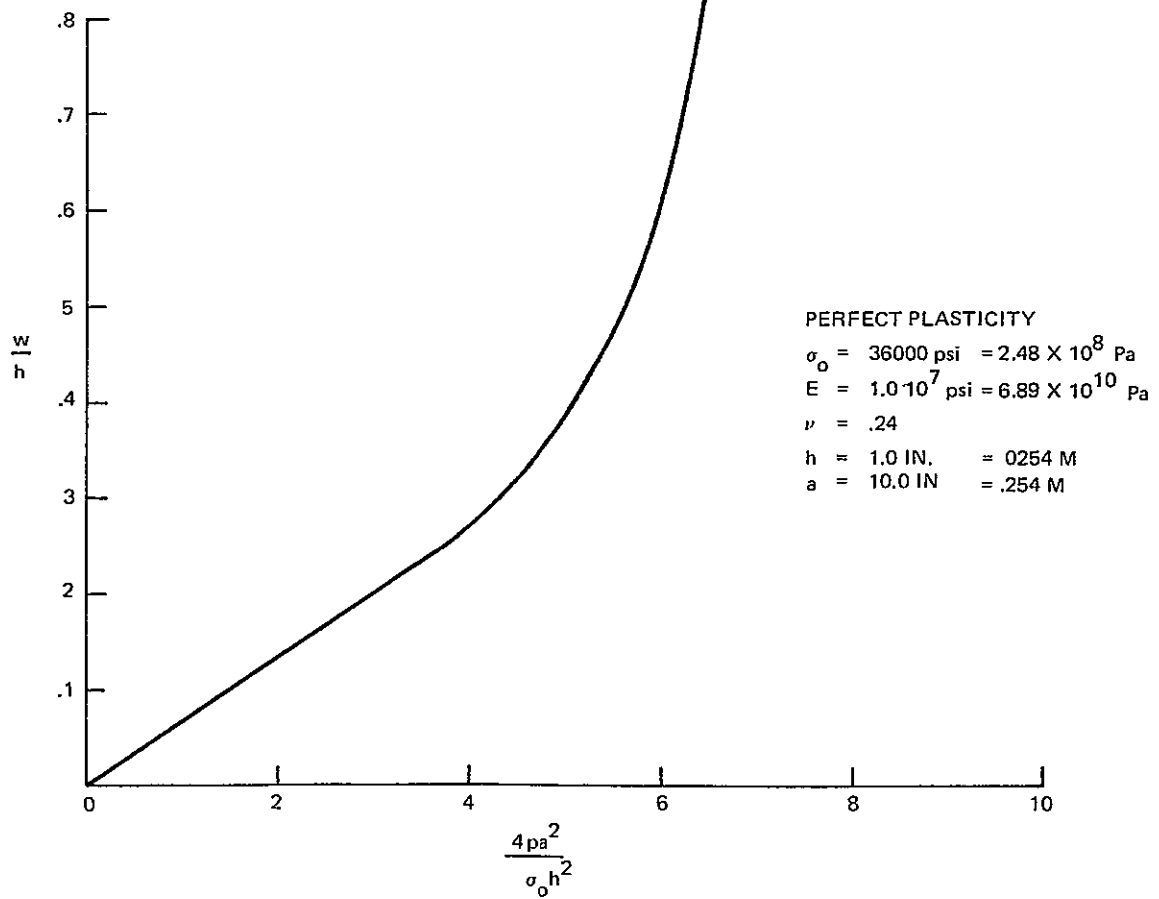
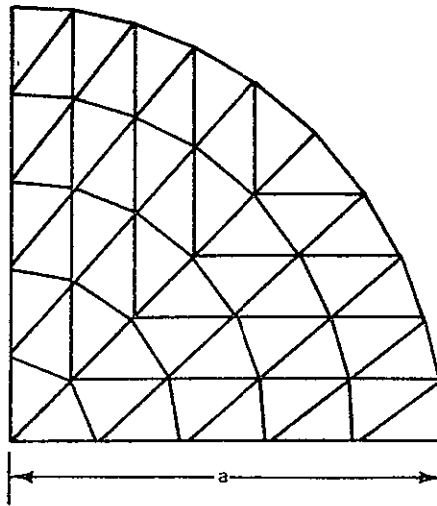
Example Problem No. 4

Program: BEND

Problem Title: Collapse of a Uniformly Loaded Circular Plate

Comments: This problem demonstrates the use of the plate element TRIP. One quadrant of a uniformly loaded, simply supported circular plate was modeled consisting of 50 elements, 36 nodes, and 151 degrees of freedom. The multipoint constraint feature was used to transform the edge rotations to their normal and tangential components in order to apply the simple support boundary conditions ($\theta_t = 0$). To be noted also is that the order of the element input was set so that the local directions of the curvatures at each node (defined by the first element containing that node) line up with the plate boundaries and lines of symmetry. This was done to enforce boundary/symmetry conditions for the higher order degrees of freedom (see Appendix IV, Section 2).

A complete discussion of the plastic analysis of this plate as well as a number of other plate configurations is given in Ref. 2. The figure shown here is for the load versus central deflection up to the collapse load.



LOAD VERSUS CENTRAL DEFLECTION

Bend Sample Problem #4 Uniformly Loaded Simply Supported Circular Plate

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM =4 UNIFORMLY LOADED CIRCULAR PLATE

1	10	1	1	50
1	-36			
TRIP		1	2	1
TRIP		2	1	4
TRIP		3	5	2
TRIP		4	2	3
TRIP		5	7	6
TRIP		6	8	7
TRIP		7	3	4
TRIP		8	4	9
TRIP		9	10	5
TRIP		10	5	6
TRIP		11	12	11
TRIP		12	6	7
TRIP		13	13	12
TRIP		14	14	13
TRIP		15	7	8
TRIP		16	15	14
TRIP		17	8	9
TRIP		18	9	16
TRIP		19	17	10
TRIP		20	10	11
TRIP		21	19	18
TRIP		22	11	12
TRIP		23	20	19
TRIP		24	12	13
TRIP		25	21	20
TRIP		26	22	21
TRIP		27	13	14
TRIP		28	23	22
TRIP		29	14	15
TRIP		30	24	23
TRIP		31	15	16
TRIP		32	16	25
TRIP		33	26	17
TRIP		34	17	18
TRIP		35	28	27
TRIP		36	18	19
TRIP		37	29	28
TRIP		38	19	20
TRIP		39	30	29
TRIP		40	20	21
TRIP		41	31	30

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

TRIP	42	32	31	21			
TRIP	43	21	22	32			
TRIP	44	33	32	22			
TRIP	45	22	23	33			
TRIP	46	34	33	23			
TRIP	47	23	24	34			
TRIP	50	25	36	35			
TRIP	48	35	34	24			
TRIP	49	24	25	35			
SEND							
0.0		1	2	5	10	17	26
1.41422		3	6	11	18	27	
2.0		4					
2.82844		7	12	19	28		
3.74162		8					
4.0		9					
4.24266		13	20	29			
5.29146		14					
5.83092		15					
6.0		16					
5.65628		21	30				
6.78232		22					
7.48328		23					
7.874		24					
8.0		25					
7.0711		31					
8.24621		32					
9.05538		33					
9.59166		34					
9.89949		35					
10.0		36					

BLANK CARD

0.0		1	4	9	16	25	36
2.0		2					
1.41422		3	8	15	24	35	
4.0		5					
3.74162		6					
2.82844		7	14	23	34		
6.0		10					
5.83092		11					
5.29146		12					
4.24266		13	22	33			
8.0		17					
7.874		18					

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

7.48328 19
6.78232 20
5.65688 21 32
10.0 26
9.89949 27
9.59166 28
9.05538 29
8.24621 30
7.07111 31

BLANK CARD

0.0 1 -36

BLANK CARD

C011110000111 1 -36
C01000000C110 1
C01100000C110 2 5 10 17
C01010000C110 4 9 16 25
C00010000001 27 -35
C00010000000 26 -36

BLANK CARD

26	4	26	5	-1.0
26	5	26	5	0.0
27	4	27	5	-.98994904
27	5	27	5	.14142199
28	4	28	5	-.95916595
28	5	28	5	.28284397
29	4	29	5	-.90553799
29	5	29	5	.42426596
30	4	30	5	-.82462101
30	5	30	5	.56568804
31	4	31	5	-.70711002
31	5	31	5	.70711002
32	4	32	5	-.56568804
32	5	32	5	.82462101
33	4	33	5	-.42426596
33	5	33	5	.90553799
34	4	34	5	-.28284397
34	5	34	5	.95916595
35	4	35	5	-.14142199
35	5	35	5	.98994904
36	4	36	5	0.0
36	5	36	5	1.0

BLANK CARD

VAT1 1.0 E+07 1.0 E+07 0.0 4.032258E+06 .24
0.0 0.0

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

	36000.0	36000.0	36000.0	20784.6	
	0.0	0.0	1.0 E+07	36000.0	36000.0
THICK	1 -50				
	1 -50				
NLRS	10 1				
SEND					
SURF	0.0	0.0	-1.0	-1.0	
	0.0	0.0	-1.0		
SEND	1 -50				
	1 2				
BLANK CARD	1 -36				
BLANK CARD	630.0	.01			
BLANK CARD					
	STOP				

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

Example Problem No. 5

Program: BEND

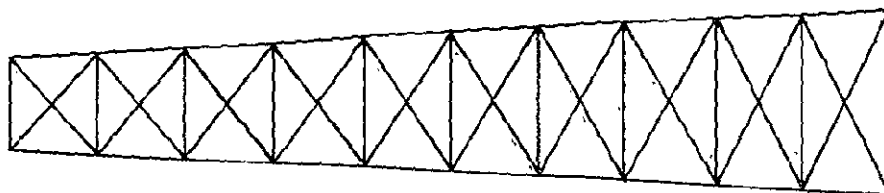
Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem demonstrates the thermal stress capability in conjunction with the triangular membrane element. To do this, an elastic plastic analysis of an annular disk subjected to a steady state axisymmetric radial temperature distribution was performed. Since the response is axisymmetric, only a slice of the disk was modeled with the multipoint constraint feature being used to enforce tangential and normal boundary conditions on one edge. Forty constant strain triangular elements connecting 32 nodes were used in the idealization. The steady state temperature distribution is applied at nodes according to the relation

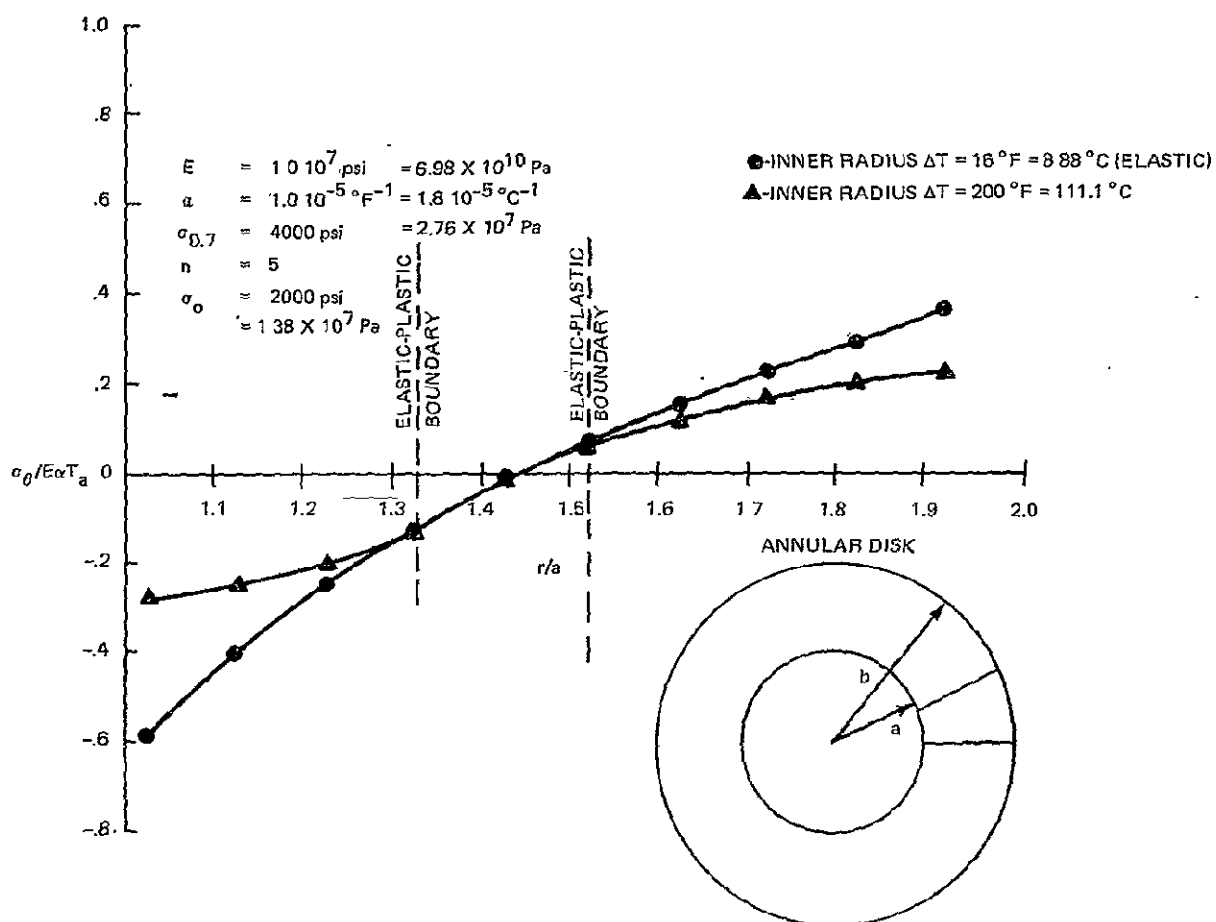
$$T = T_a - (T_b - T_a) \ln(r/a)/\ln(b/a)$$

where T_a and T_b are the inner and outer radius temperature, respectively.

Results shown in the figure are for dimensionless circumferential stress versus the radial coordinate. Comparison between these results and Ref. 4 are good.



FINITE ELEMENT MODEL OF A SECTOR OF THE DISK



DISTRIBUTION OF CIRCUMFERENTIAL STRESS

Bend Sample Problem #5 Thermoplastic Analysis of Circular Disk

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM #5 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK

	0	5	1	-9	13	-15	19	-21	25	-27	31	-33	37	-39	43	-45
	1	-3	7	-9	13	-15	19	-21	25	-27	31	-33	37	-39	43	-45
49	-51	55	-57	61	63											
TRIM		01	01	0	03	0	02									
TRIM		02	03	0	09	0	02									
TRIM		03	09	0	07	0	02									
TRIM		04	07	0	01	0	02									
TRIM		05	07	0	09	0	08									
TRIM		06	09	0	15	0	08									
TRIM		07	15	0	13	0	08									
TRIM		08	13	0	07	0	08									
TRIM		09	13	0	15	0	14									
TRIM		10	15	0	21	0	14									
TRIM		11	21	0	19	0	14									
TRIM		12	19	0	13	0	14									
TRIM		13	19	0	21	0	20									
TRIM		14	21	0	27	0	20									
TRIM		15	27	0	25	0	20									
TRIM		16	25	0	19	0	20									
TRIM		17	25	0	27	0	26									
TRIM		18	27	0	33	0	26									
TRIM		19	33	0	31	0	26									
TRIM		20	31	0	25	0	26									
TRIM		21	31	0	33	0	32									
TRIM		22	33	0	39	0	32									
TRIM		23	39	0	37	0	32									
TRIM		24	37	0	31	0	32									
TRIM		25	37	0	39	0	38									
TRIM		26	39	0	45	0	38									
TRIM		27	45	0	43	0	38									
TRIM		28	43	0	37	0	38									
TRIM		29	43	0	45	0	44									
TRIM		30	45	0	51	0	44									
TRIM		31	51	0	49	0	44									
TRIM		32	49	0	43	0	44									
TRIM		33	49	0	51	0	50									
TRIM		34	51	0	57	0	50									
TRIM		35	57	0	55	0	50									
TRIM		36	55	0	49	0	50									
TRIM		37	55	0	57	0	56									
TRIM		38	57	0	63	0	56									
TRIM		39	63	0	61	0	56									
TRIM		40	61	0	55	0	56									

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

SEND

1.0	E 00	3	6
1.1	E 00	9	12
1.2	E 00	15	18
1.3	E 00	21	24
1.4	E 00	27	30
1.5	E 00	33	36
1.6	E 00	39	42
1.7	E 00	45	48
1.8	E 00	51	54
1.9	E 00	57	60
2.0	E 00	63	66
.99452189	E 00	1	4
1.0939741	E 00	7	10
1.1934262	E 00	13	16
1.2928784	E 00	19	22
1.3923306	E 00	25	28
1.4917828	E 00	31	34
1.5912350	E 00	37	40
1.6814254	E 00	43	46
1.7901394	E 00	49	52
1.8895916	E 00	55	58
1.9890437	E 00	61	64
1.05	E 00	2	5
1.15	E 00	8	11
1.25	E 00	14	17
1.35	E 00	20	23
1.45	E 00	26	29
1.55	E 00	32	35
1.65	E 00	38	41
1.75	E 00	44	47
1.85	E 00	50	53
1.95	E 00	56	59

BLANK CARD

0.0	E 00	3	-3	-66
.0523359	E 00	2	5	
.0575695	E 00	8	11	
.0628031	E 00	14	17	
.0680367	E 00	20	23	
.0732703	E 00	26	29	
.0785039	E 00	32	35	
.0837375	E 00	38	41	
.0889711	E 00	44	47	
.0942047	E 00	50	53	

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

.0994383 E 00 56 59
.1046712 E 00 62 65
.1045284 E 00 1 4
.1149813 E 00 7 10
.1254341 E 00 13 16
.1358870 E 00 19 22
.1463398 E 00 25 28
.1567926 E 00 31 34
.1672455 E 00 37 40
.1776983 E 00 43 46
.1881512 E 00 49 52
.1986040 E 00 55 58
.2090569 E 00 61 64

BLANK CARD

0.0 1 -63

BLANK CARD

110000000000 1 -3 7 -9 13 -15 19 -21 25 -27 31 -33 37
-39 43 -45 49 -51 55 -57 61 63
100000000000 1 -6 -61 3 -6 -63

BLANK CARD

1	1	1	1	.9945218	1	2	-.1045284
1	2	1	1	.1045284	1	2	.9945218
7	1	7	1	.9945218	7	2	-.1045284
7	2	7	1	.1045284	7	2	.9945218
13	1	13	1	.9945218	13	2	-.1045284
13	2	13	1	.1045284	13	2	.9945218
19	1	19	1	.9945218	19	2	-.1045284
19	2	19	1	.1045284	19	2	.9945218
25	1	25	1	.9945218	25	2	-.1045284
25	2	25	1	.1045284	25	2	.9945218
31	1	31	1	.9945218	31	2	-.1045284
31	2	31	1	.1045284	31	2	.9945218
37	1	37	1	.9945218	37	2	-.1045284
37	2	37	1	.1045284	37	2	.9945218
43	1	43	1	.9945218	43	2	-.1045284
43	2	43	1	.1045284	43	2	.9945218
49	1	49	1	.9945218	49	2	-.1045284
49	2	49	1	.1045284	49	2	.9945218
55	1	55	1	.9945218	55	2	-.1045284
55	2	55	1	.1045284	55	2	.9945218
61	1	61	1	.9945218	61	2	-.1045284
61	2	61	1	.1045284	61	2	.9945218

BLANK CARD

MAT1 1.0 E 07 1.0 E 07 0.0 F 00 3.846153 E 06 .30000E 00

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

1.0 E-05 1.0 E-05
 2.0 E 03 2.0 E 03 2.0 E 03 1.1547002E 03
 5.0 E CC 4.0 F 03 1.0 E 07 2.0 E 03 2.0 E 03

1 -40
 THIK .1
 1 -40

SEND
 TPU

1.0	1	3
.8624964	7	9
.7369055	13	15
.6214883	19	21
.5145732	25	27
.4150375	31	33
.3219281	37	39
.2344652	43	45
.1520030	49	51
.07400054	55	57
.0	61	63
.92961067	2	
.79836614	8	
.6780714	14	
.56704059	20	
.46394710	26	
.36773178	32	
.27753398	38	
.19264508	44	
.11247473	50	
.036525879	56	

BLANK CARD

SEND
 1 -40

BLANK CARD

1 -63

BLANK CARD

200.0 .05

BLANK CARD

STOP

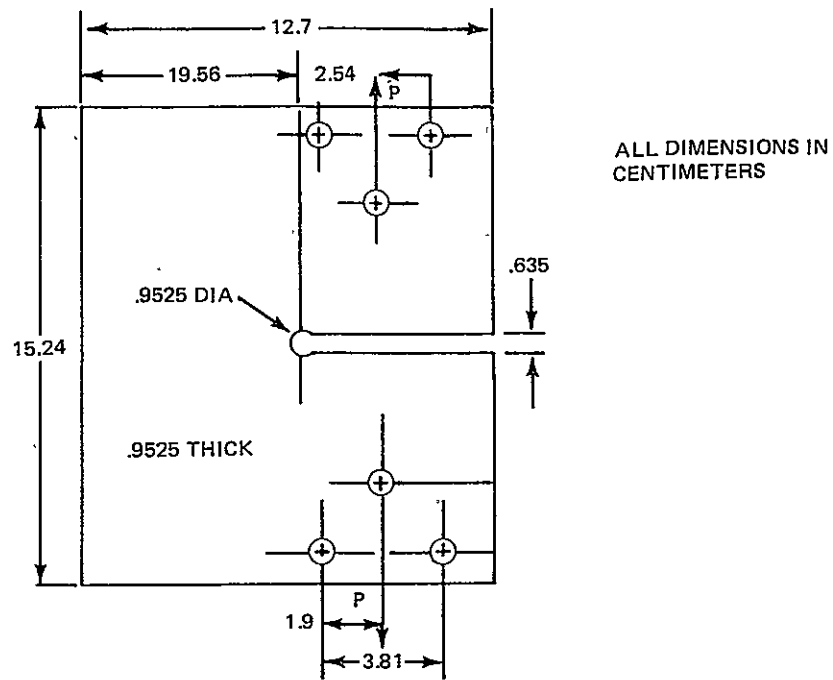
.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

Example Problem No. 6

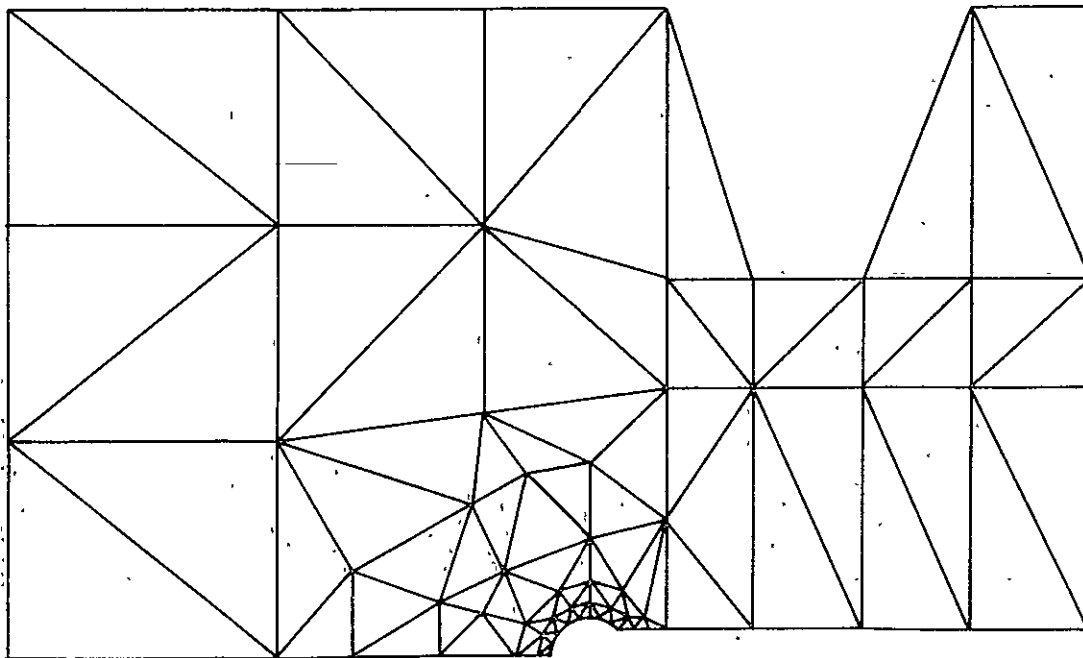
Program: BEND

Problem Title: Elastic Plastic Analysis of an SAE Keyhole Specimen

Comments: The accompanying figures show an SAE keyhole specimen, the finite element idealization of half of the plate, and results for the strain at the notch versus load. The idealization uses 92 constant strain elements connecting 64 nodes that leads to 120 degrees of freedom.

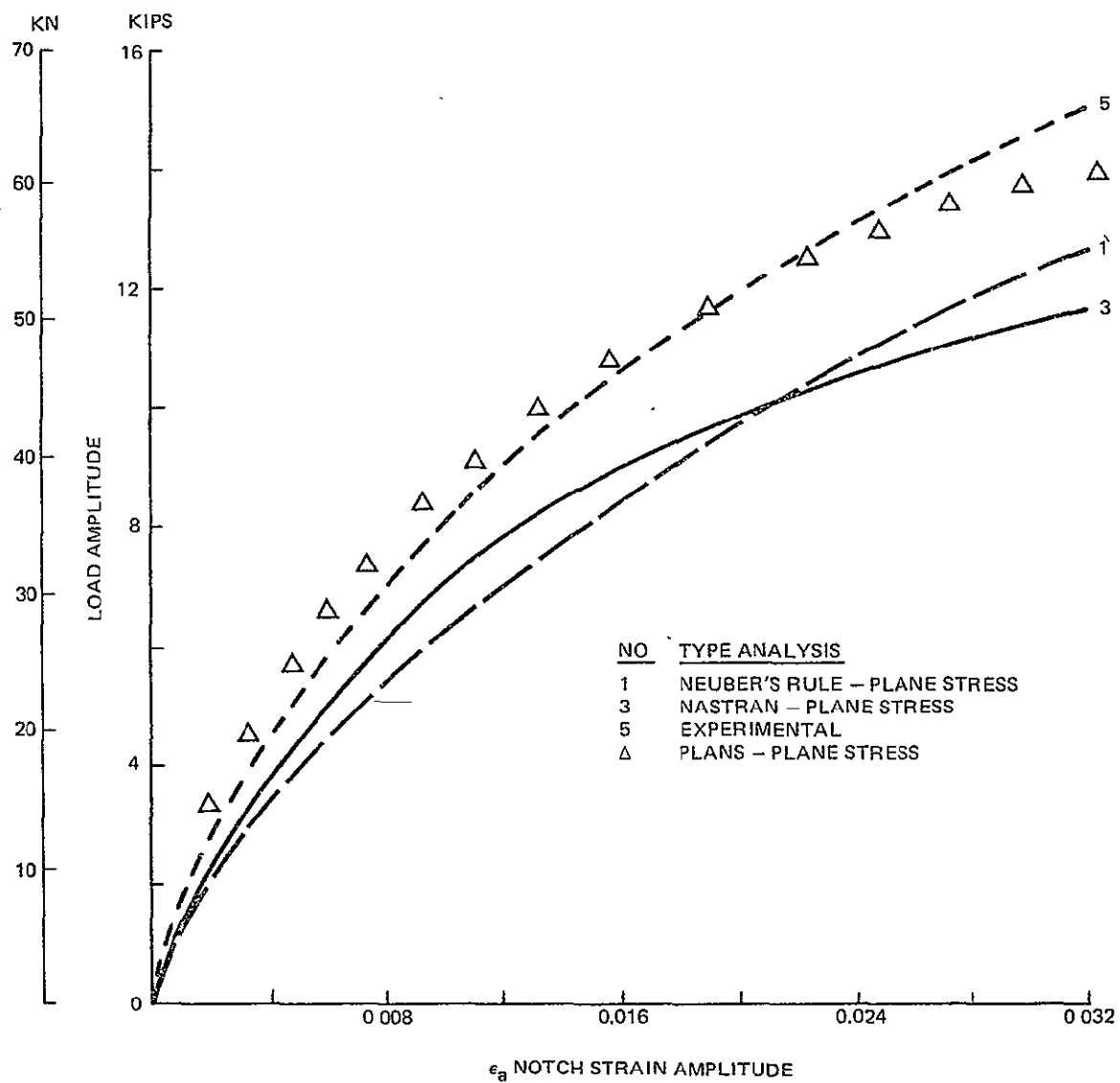


SAE KEYHOLE SPECIMEN



FINITE ELEMENT IDEALIZATION

Bend Sample Problem # 6 Load versus Notch Strain for SAE Keyhole Specimen



Bend Sample Problem #6 (Continued)

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM # 6 SAE KEYHOLE SPECIMEN

1	0	1	1	50
1	-64			
GLXY				
TRIM	1	1	2	10
TRIM	2	2	3	10
TRIM	3	3	12	10
TRIM	4	10	12	11
TRIM	5	11	12	19
TRIM	6	20	19	12
TRIM	7	13	20	12
TRIM	8	21	20	13
TRIM	9	22	21	13
TRIM	10	3	13	12
TRIM	11	3	4	13
TRIM	12	4	5	13
TRIM	13	5	14	13
TRIM	14	14	22	13
TRIM	15	15	22	14
TRIM	16	6	15	14
TRIM	17	5	6	14
TRIM	18	25	6	5
TRIM	19	25	27	6
TRIM	20	26	27	25
TRIM	21	27	28	6
TRIM	22	28	7	6
TRIM	23	6	16	15
TRIM	24	6	7	16
TRIM	25	7	17	16
TRIM	26	16	17	23
TRIM	27	17	18	23
TRIM	28	18	24	23
TRIM	29	8	18	17
TRIM	30	8	9	18
TRIM	31	8	30	9
TRIM	32	8	29	30
TRIM	33	28	29	7
TRIM	34	7	29	8
TRIM	35	7	8	17
TRIM	36	25	5	64
TRIM	37	64	5	4
TRIM	38	63	64	4
TRIM	39	62	63	4
TRIM	40	62	4	3

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

TRIM	41	61	62	3
TRIM	42	2	61	3
TRIM	43	2	59	61
TRIM	44	59	60	61
TRIM	45	60	62	61
TRIM	46	60	57	62
TRIM	47	57	63	62
TRIM	48	57	58	63
TRIM	49	58	64	63
TRIM	50	58	25	64
TRIM	51	59	55	60
TRIM	52	55	56	60
TRIM	53	56	57	60
TRIM	54	56	49	57
TRIM	55	55	48	56
TRIM	56	48	49	56
TRIM	57	49	50	57
TRIM	58	50	58	57
TRIM	59	50	51	58
TRIM	60	51	52	58
TRIM	61	52	25	58
TRIM	62	52	53	25
TRIM	63	53	26	25
TRIM	64	48	40	41
TRIM	65	48	41	49
TRIM	66	41	42	49
TRIM	67	49	42	50
TRIM	68	42	43	50
TRIM	69	43	51	50
TRIM	70	43	44	51
TRIM	71	44	45	51
TRIM	72	45	52	51
TRIM	73	45	46	52
TRIM	74	46	53	52
TRIM	75	47	53	46
TRIM	76	47	54	53
TRIM	77	54	26	53
TRIM	78	40	31	32
TRIM	79	40	32	41
TRIM	80	32	33	41
TRIM	81	41	33	42
TRIM	82	33	34	42
TRIM	83	42	34	43
TRIM	84	34	35	43

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

TRIM	85	35	44	43
TRIM	86	35	36	44
TRIM	87	36	37	44
TRIM	88	37	45	44
TRIM	89	37	38	45
TRIM	90	38	46	45
TRIM	91	38	39	46
TRIM	92	39	47	46
SEND				

0.0000E	00	1
1.2500E	00	2
1.2500E	00	3
2.2000E	00	4
3.0500E	00	5
3.4500E	00	6
3.9500E	00	7
4.4500E	00	8
5.0000E	00	9
0.0000E	00	10
0.0000E	00	11
1.2500E	00	12
2.2000E	00	13
3.0500E	00	14
3.4500E	00	15
3.9500E	00	16
4.4500E	00	17
5.0000E	00	18
0.0000E	00	19
1.2500E	00	20
2.2000E	00	21
3.0500E	00	22
4.4500E	00	23
5.0000E	00	24
3.0500E	00	25
3.0500E	00	26
3.4500E	00	27
3.9500E	00	28
4.4500E	00	29
5.0000E	00	30
2.5125E	00	31
2.5217E	00	32
2.5483E	00	33
2.5900E	00	34
2.6421E	00	35

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

2.7000E 00 36
2.7579E 00 37
2.8102E 00 38
2.8375E 00 39
2.4500E 00 40
2.4750E 00 41
2.5250E 00 42
2.6000E 00 43
2.7000E 00 44
2.8000E 00 45
2.8750E 00 46
2.9000E 00 47
2.3500E 00 48
2.4000E 00 49
2.5500E 00 50
2.7000E 00 51
2.8500E 00 52
2.9500E 00 53
2.9750E 00 54
2.0000E 00 55
2.2000E 00 56
2.3000E 00 57
2.7000E 00 58
1.6000E 00 59
2.0000E 00 60
1.6000E 00 61
2.1500E 00 62
2.4000E 00 63
2.7000E 00 64

0.0000E 00 1
0.0000E 00 2
1.0000E 00 3
1.1250E 00 4
1.2500E 00 5
1.2500E 00 6
1.2500E 00 7
1.2500E 00 8
1.2500E 00 9
1.0000E 00 10
2.0000E 00 11
2.0000E 00 12
2.0000E 00 13
1.7500E 00 14

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

BLANK CARD

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

1.7500E 00 15
 1.7500E 00 16
 1.7500E 00 17
 1.7500E 00 18
 3.0000E 00 19
 3.0000E 00 20
 3.0000E 00 21
 3.0000E 00 22
 3.0000E 00 23
 3.0000E 00 24
 0.6250E 00 25
 0.1250E 00 26
 0.1250E 00 27
 0.1250E 00 28
 0.1250E 00 29
 0.1250E 00 30
 0.9000E 00 31
 0.0579E 00 32
 0.1102E 00 33
 0.1517E 00 34
 0.1783E 00 35
 0.1875E 00 36
 0.1783E 00 37
 0.1517E 00 38
 0.1250E 00 39
 0.0000E 00 40
 0.1000E 00 41
 0.1750E 00 42
 0.2250E 00 43
 0.2500E 00 44
 0.2250E 00 45
 0.1750E 00 46
 0.1250E 00 47
 0.0000E 00 48
 0.1500E 00 49
 0.3000E 00 50
 0.3500E 00 51
 0.3000E 00 52
 0.1750E 00 53
 0.1250E 00 54
 0.0000E 00 55
 0.2000E 00 56
 0.4000E 00 57
 0.5500E 00 58

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

0.0000E 00 59
0.2500E 00 60
0.4000E 00 61
0.7000E 00 62
0.8500E 00 63
0.9000E 00 64
BLANK CARD
0.0000E 00 1 -64
BLANK CARD
110000000000 1 -64
000000000000 1
100000000000 2 31 40 48 55 59
BLANK CARD
BLANK CARD
MAT1 29.5000E 06 29.5000E 06 0.0000E 00 1.134620E 07 0.3000E 00
0.0000E 00 0.0000E 00
35.000E 03 35.000E 03 35.000E 03 2.02070E 04
5.26320E 00 38.5890E 03 29.5000E 06 35.000E 03 35.0000E 03
1 -92
TH1K 0.3750E 00
1 -92
SFND
CUNC
15 0.0000E 00 -1.000E 00 0.0000E 00
0.0000E 00 0.000E 00 0.000E 00
16 0.0000E 00 -1.000E 00 0.000E 00
0.000E 00 0.000E 00 0.000E 00
BLANK CARD
SFND
1 -92
BLANK CARD
1 -64
BLANK CARD
-7.5000E 03 0.0100E 00
BLANK CARD
STOP

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

[illegible][illegible]

A large graphic of the number 9, formed by the letter 'S'. The 'S' characters are arranged in a grid-like pattern to create the shape of the digit. The top horizontal bar is composed of 10 'S' characters. The vertical stem on the right is composed of 10 'S' characters. The bottom horizontal bar is composed of 10 'S' characters. The left vertical stem is composed of 10 'S' characters. The internal curves are filled with 'S' characters to complete the shape of the number 9.

[illegible]

REVB Y MODULE

ELASTIC-PLASTIC ANALYSIS

PREPARED BY

THE RESEARCH DEPARTMENT
OF GRUMMAN AEROSPACE CORPORATION
UNDER PARTIAL FUNDING FROM
THE NASA LANGLEY RESEARCH CENTER

Example Problem No. 7

Program: REVBY

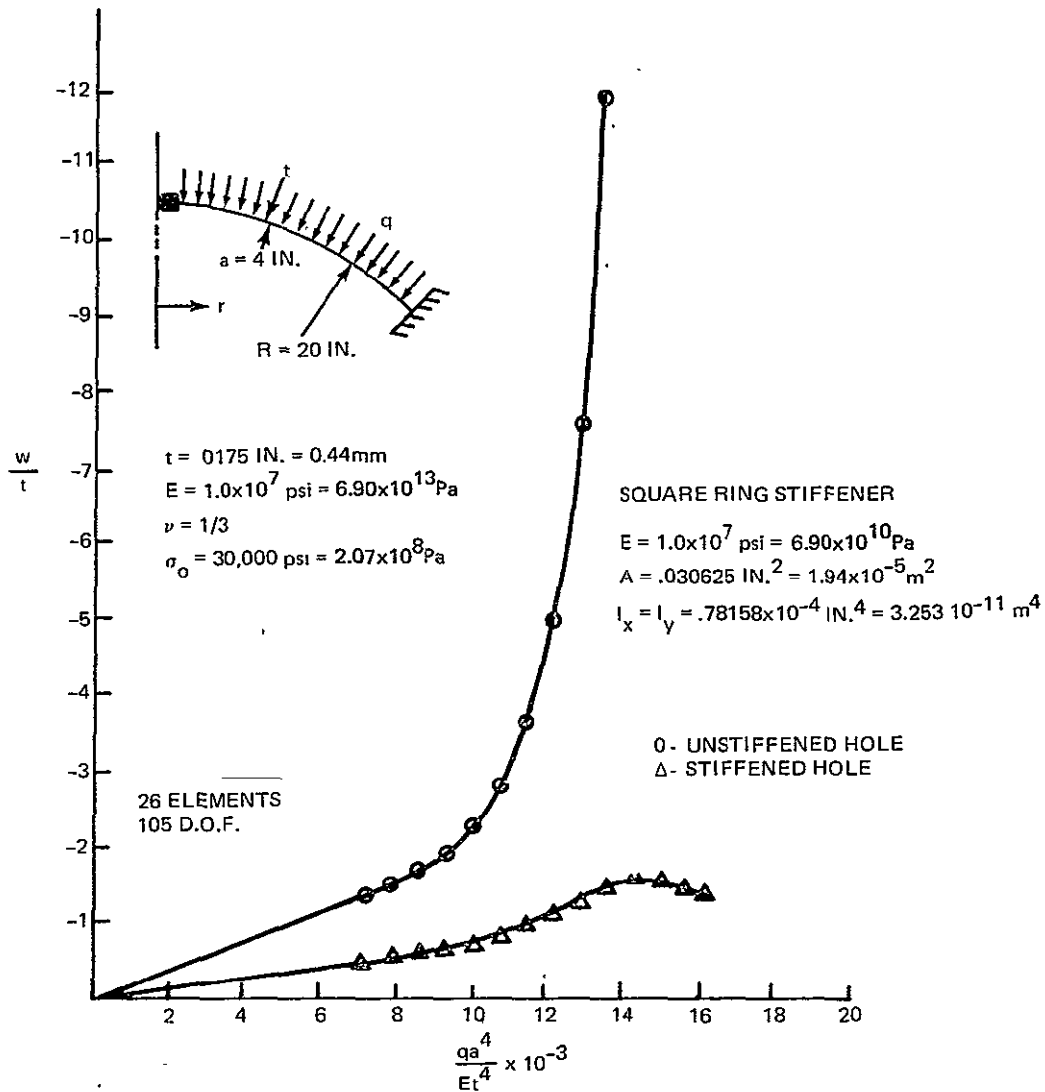
Problem Title: Plastic Analysis of a Ring Stiffened Spherical Shell

Comments: This problem demonstrates the use of the shell and ring element. The structure consists of a uniformly loaded spherical shell with a stiffened circular hole at the shell apex. The problem is idealized using 26 axisymmetric shell elements and one ring element connecting 27 nodes.

The figure shows the normal displacement versus applied pressure at the ring shell interface and also results for an unstiffened shell. Sudden collapse occurs for the stiffened shell at $qa^4/Et^4 = 15000$.

Additional results for this problem are in Ref. 3.

LOAD-DEFLECTION CURVES FOR RING-STIFFENED
SPHERICAL SHELL UNDER EXTERNAL PRESSURE



(a) NORMAL DISPLACEMENT AT THE HOLE BOUNDARY

REVBY Sample Problem #1 Ring Stiffened Spherical Shell

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

REVBY SAMPLE PROBLEM =1 RING STIFFENED SPHERICAL SHELL

1 10
1 -27
SHELL 1 1 2
SHELL 2 2 3
SHELL 3 3 4
SHELL 4 4 5
SHELL 5 5 6
SHELL 6 6 7
SHELL 7 7 8
SHELL 8 8 9
SHELL 9 9 10
SHELL 10 10 11
SHELL 11 11 12
SHELL 12 12 13
SHELL 13 13 14
SHELL 14 14 15
SHELL 15 15 16
SHELL 16 16 17
SHELL 17 17 18
SHELL 18 18 19
SHELL 19 19 20
SHELL 20 20 21
SHELL 21 21 22
SHELL 22 22 23
SHELL 23 23 24
SHELL 24 24 25
SHELL 25 25 26
SHELL 26 26 27
RING 27 1
SEND

3.9999998E-01 1
4.8724484E-01 2
5.7448047E-01 3
6.6170520E-01 4
7.4891728E-01 5
8.6517763E-01 6
9.8140872E-01 7
1.0976057E 00 8
1.2834435E 00 9
1.4691706E 00 10
1.6547689E 00 11
1.8402252E 00 12
2.0255213E 00 13

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

2.2106419E 00 14
 2.3955717E 00 15
 2.5802937E 00 16
 2.7647915E 00 17
 2.9490499E 00 18
 3.1330528E 00 19
 3.3167849E 00 20
 3.4314718E 00 21
 3.5460434E 00 22
 3.6604939E 00 23
 3.7462511E 00 24
 3.8319378E 00 25
 3.9175501E 00 26
 4.0030890E 00 27

BLANK CARD

-4.0003993E-03 1
 -5.9360676E-03 2
 -8.2523972E-03 3
 -1.0949340E-02 4
 -1.4026847E-02 5
 -1.8722069E-02 6
 -2.4093587E-02 7
 -3.0141219E-02 8
 -4.1223209E-02 9
 -5.4034557E-02 10
 -6.8574131E-02 11
 -8.4840655E-02 12
 -1.0283279E-01 13
 -1.2254894E-01 14
 -1.4398742E-01 15
 -1.6714633E-01 16
 -1.9202363E-01 17
 -2.1861726E-01 18
 -2.4692488E-01 19
 -2.7694398E-01 20
 -2.9657394E-01 21
 -3.1667075E-01 22
 -3.3783376E-01 23
 -3.5399282E-01 24
 -3.7052590E-01 25
 -3.8743265E-01 26
 -4.0471286E-01 27

BLANK CARD

2.0001329E-02 1

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

2.4364647E-02 2
 2.8727978E-02 3
 3.3091299E-02 4
 3.7454616E-02 5
 4.3272387E-02 6
 4.9090147E-02 7
 5.4907918E-02 8
 6.4216316E-02 9
 7.3524714E-02 10
 8.2833171E-02 11
 9.2141569E-02 12
 1.0145003E-01 13
 1.1075842E-01 14
 1.2006682E-01 15
 1.2937528E-01 16
 1.3868368E-01 17
 1.4799213E-01 18
 1.5730053E-01 19
 1.6660893E-01 20
 1.7242670E-01 21
 1.7824447E-01 22
 1.8406224E-01 23
 1.8842554E-01 24
 1.9278890E-01 25
 1.9715220E-01 26
 2.0151556E-01 27

7-43

BLANK CARD

111100 1 -26
 000100 27

BLANK CARD

BLANK CARD

1.0000000E 07 1.0000000E 07 0.3750000E 07 0.3333333E 00
 0.3000000E 05 0.3000000E 05 0.3000000E 05 0.1732000E 05 0.0000000E 00
 0.0000000E 00 0.3000000E 05 1.0000000E 07
 1.0000000E-05 1.0000000E-05
 1 -26

BLANK CARD

1.7499998E-02
 1 -27

BLANK CARD

6 10
 1.0E 07 3.0625E-02 7.8153E-05 0.0000000E 00 7.8158E-05
 0.0E 00 3.75E-02

SPEC . .175E00 .175E00

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPJT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

4.0E03 0.0E00 0.0E00
 1.0000000E-05
 27

BLANK CARD

0.0000000E 00 -1.0000000E 00 0.0000000E 00
 1 -27

BLANK CARD

BLANK CARD

SEND
 -27

BLANK CARD

-27

BLANK CARD

26 27

BLANK CARD

-27

BLANK CARD

60.0 .01

BLANK CARD

STOP

7-4/4

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

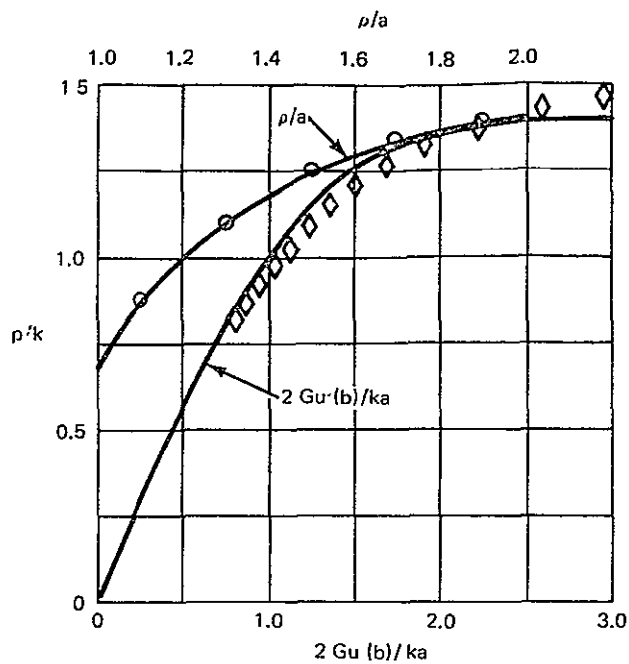
Example Problem No. 8

Program: REVBY

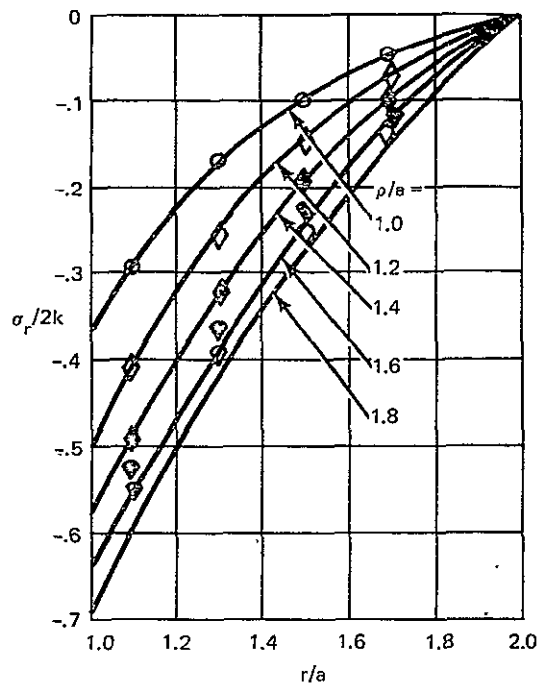
Problem Title: Thick Tube Under Internal Pressure

Comments: This problem demonstrates the use of the axisymmetric revolved triangular element, TRIR. Five bays of four triangles each (20) are used to idealize thick tube under internal pressure.

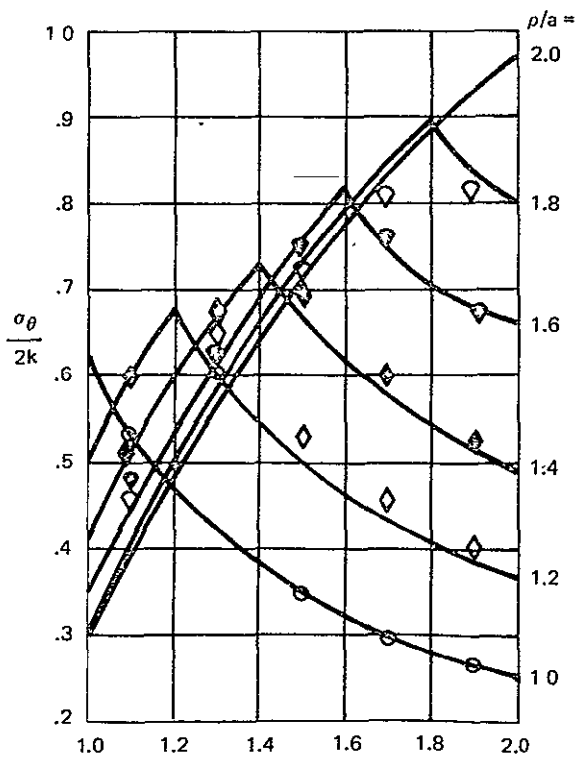
A series of results are shown in the figure and compared with a solution in Ref. 5. Additional discussion of this problem is in Ref. 3.



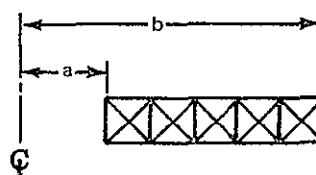
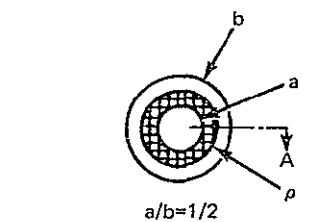
PRESSURE p VERSUS RADIAL DISPLACEMENT $u(b)$ AND RADIUS OF ELASTIC-PLASTIC BOUNDARY, ρ



DISTRIBUTION OF RADIAL STRESS



DISTRIBUTION OF CIRCUMFERENTIAL STRESS



SECT A
28 d.o.f. IDEALIZATION
REF. (5)

○ ◇ ◆ ▼ FINITE-ELEMENT SOLUTION
 k = YIELD STRESS IN SHEAR

REVBY Sample Problem # 2 Thick Cylinder Under Internal Pressure

[illegible]

	-5	1-1	-105	7	-12
TRE		1	1	101	12
TRE		2	101	8	7
TRE		3	101	2	x
TRE		4	1	2	101
TRE		5	2	102	8
TRE		6	102	9	8
TRE		7	3	9	102
TRE		8	2	3	102
TRE		9	3	103	9
TRE		10	103	10	9
TRE		11	103	4	103
TRE		12	3	4	103
TRE		13	4	104	10
TRE		14	104	11	10
TRE		15	104	5	11
TRE		16	4	5	104
TRE		17	5	105	11
TRE		18	105	12	11
TRE		19	105	6	12
TRE		20	5	6	105

$$\begin{array}{r} 100 \ 100 \\ 1100 \ 20 \end{array} \quad \begin{array}{r} 1 \\ 11 \end{array} = 105 \quad \begin{array}{r} 7 \\ -12 \end{array}$$

BLANK CARD
BLANK CARD

ORIGINAL PAGE IS
OF POOR QUALITY

[illegible]

123456780¹ 123456780² 123456780³ 123456780⁴ 123456780⁵ 123456780⁶ 123456780⁷ 123456780⁸

BLANK CARD

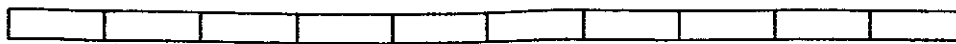
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Example Problem No. 9

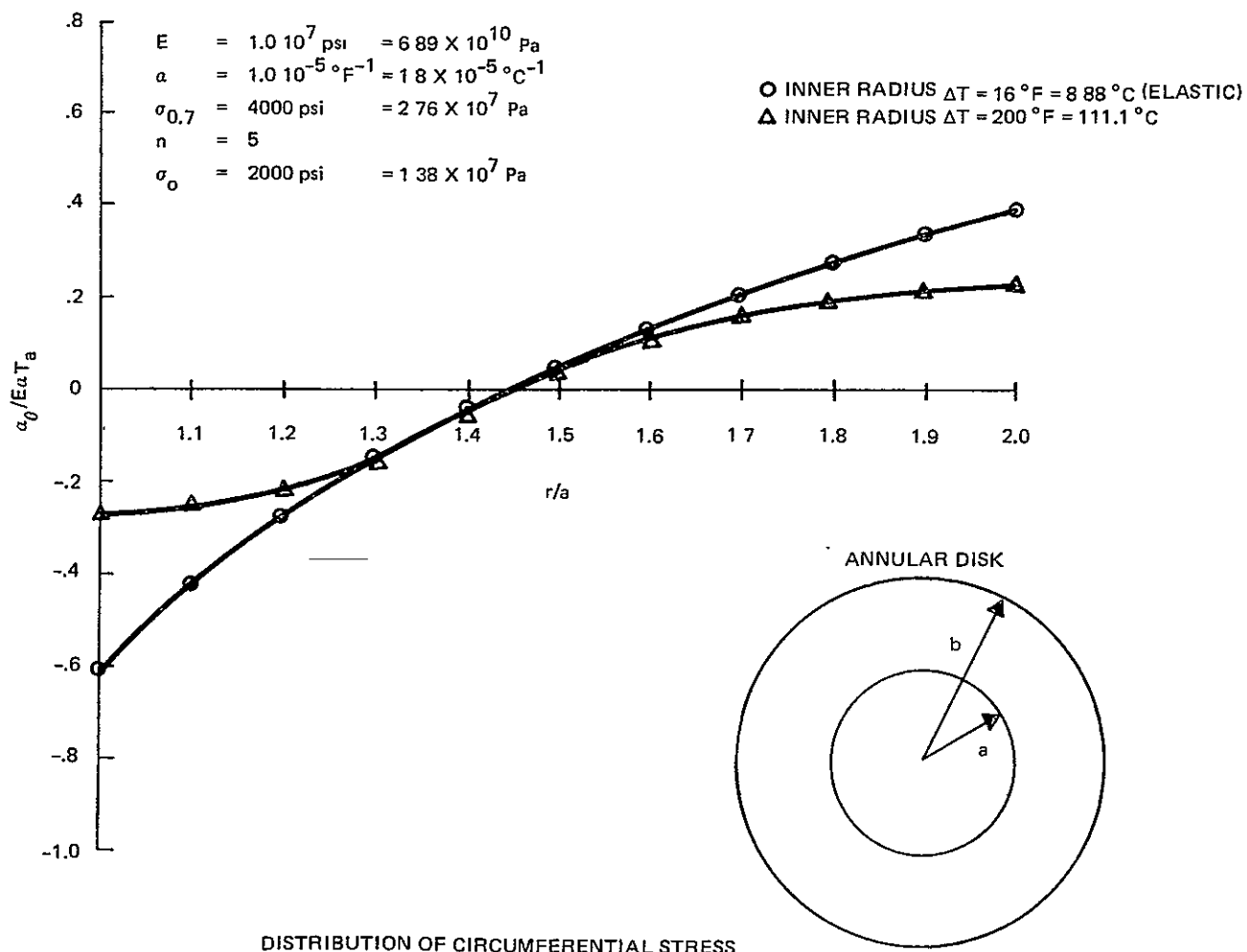
Program: REVBY

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem is the same as sample problem 5 using the BEND program. Here 10 shell elements are used connecting 11 nodes leading to 22 degrees of freedom. Comparison made with the results of Ref. 4 indicate excellent agreement.



FINITE ELEMENT IDEALIZATION USING SHELL ELEMENTS



REVBY Sample Problem # 3 Thermoplastic Analysis of a Circular Disk Shell Elements

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

REVBY SAMPLE PROBLEM # 3 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK - II

0 10
 1 -11
 SHEL 1 1 2
 SHEL 2 2 3
 SHEL 3 3 4
 SHEL 4 4 5
 SHEL 5 5 6
 SHEL 6 6 7
 SHEL 7 7 8
 SHEL 8 8 9
 SHEL 9 9 10
 SHEL 10 10 11
 SEND

1.0000000E 00 1
 1.1000000E 00 2
 1.2000000E 00 3
 1.3000000E 00 4
 1.4000000E 00 5
 1.5000000E 00 6
 1.6000000E 00 7
 1.7000000E 00 8
 1.8000000E 00 9
 1.9000000E 00 10
 2.0000000E 00 11

BLANK CARD
 0.0000000E 00 1 -11

BLANK CARD
 0.0000000E 00 1 -11

BLANK CARD
 100100 1 -11

BLANK CARD

BLANK CARD
 10.0000000E 06 10.0000000E 06 0.3840153E 07 0.3000000E 00
 2.0000000E 03 2.0000000E 03 2.0000000E 03 1.1547005E 03 0.5000000E 01
 4.0000000E 03 2.0000000E 03 10.0000000E 06
 1.0000000E-05 1.0000000E-05
 1 -10

BLANK CARD

0.1000000E 00
 1 -11

BLANK CARD

0 10

BLANK CARD

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

```

      0.0000000E 00
      1.0000000E 00      1.0000000E 00      1.0000000E 00
1      8.6249648E-01      8.6249648E-01      8.6249648E-01
2      7.3696559E-01      7.3696559E-01      7.3696559E-01
3      5.1457317E-01      5.1457317E-01      5.1457317E-01
5      4.1503750E-01      4.1503750E-01      4.1503750E-01
6      3.2192809E-01      3.2192809E-01      3.2192809E-01
7      2.3445525E-01      2.3445525E-01      2.3445525E-01
8      1.5200309E-01      1.5200309E-01      1.5200309E-01
9      7.4000581E-02      7.4000581E-02      7.4000581E-02
10     0.0000000E 00      0.0000000E 00      0.0000000E 00
11     6.2148838E-01      6.2148838E-01      6.2148838E-01

```

SEND

-11

BLANK CARD

-12

BLANK CARD

-11

BLANK CARD

-12

BLANK CARD

200.0

.050

BLANK CARD

STOP

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

REVBY SAMPLE PROBLEM =4 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK - I

0 10
 1 -32

TRIR	1	1	23	12
TRIR	2	1	2	23
TRIR	3	2	13	23
TRIR	4	13	12	23
TRIR	5	2	24	13
TRIR	6	2	3	24
TRIR	7	3	14	24
TRIR	8	14	13	24
TRIR	9	3	25	14
TRIR	10	3	4	25
TRIR	11	4	15	25
TRIR	12	15	14	25
TRIR	13	4	26	15
TRIR	14	4	5	26
TRIR	15	5	16	26
TRIR	16	16	15	26
TRIR	17	5	27	16
TRIR	18	5	6	27
TRIR	19	6	17	27
TRIR	20	17	16	27
TRIR	21	6	28	17
TRIR	22	6	7	28
TRIR	23	7	18	28
TRIR	24	18	17	28
TRIR	25	7	29	18
TRIR	26	7	8	29
TRIR	27	8	19	29
TRIR	28	19	18	29
TRIR	29	8	30	19
TRIR	30	8	9	30
TRIR	31	9	20	30
TRIR	32	20	19	30
TRIR	33	9	31	20
TRIR	34	9	10	31
TRIR	35	10	21	31
TRIR	36	21	20	31
TRIR	37	10	32	21
TRIR	38	10	11	32
TRIR	39	11	22	32
TRIR	40	22	21	32
SEND				

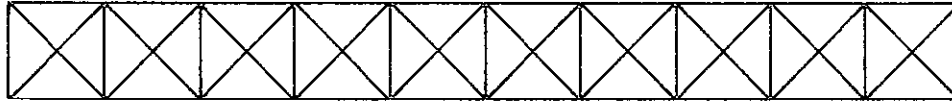
.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

Example Problem No. 10

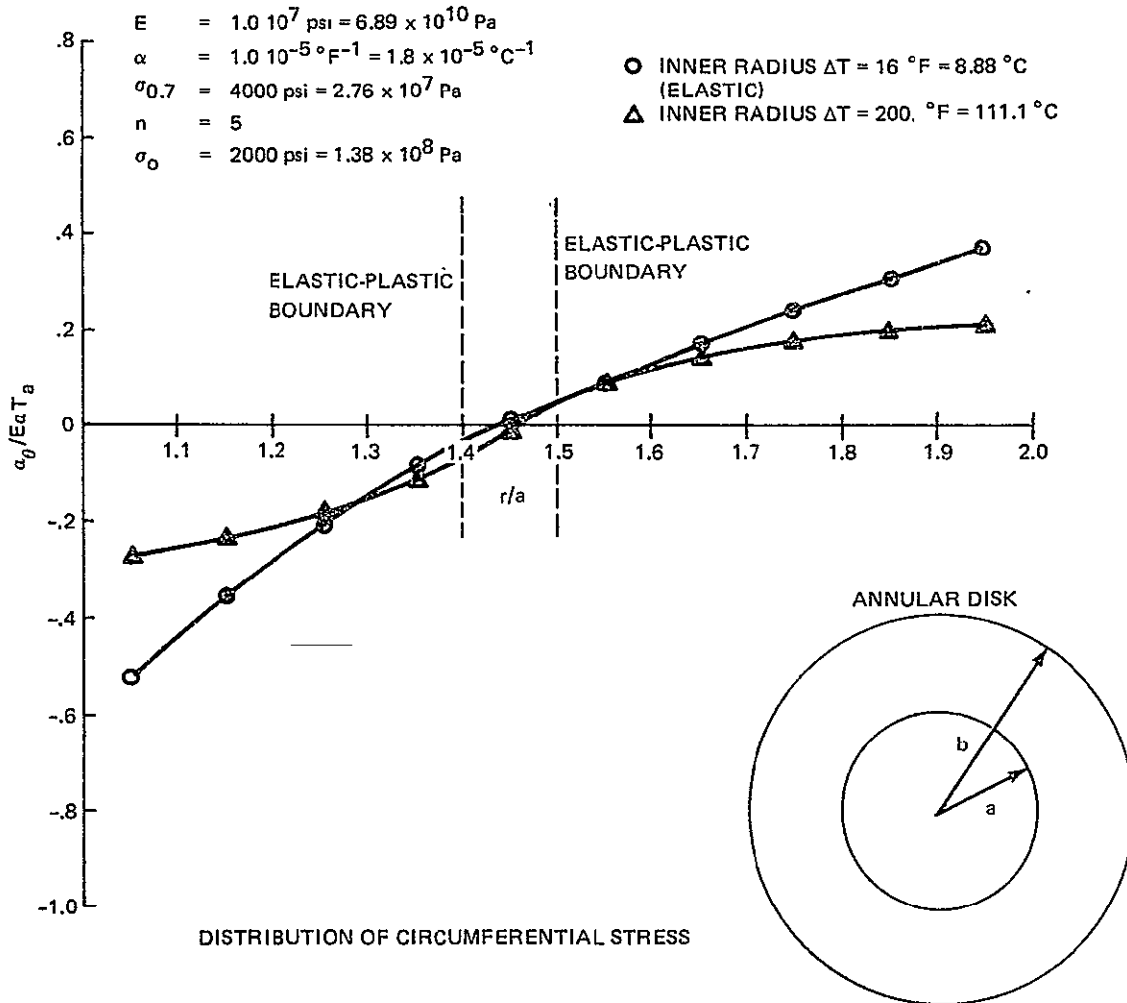
Program: REVBY

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: Same as sample problem 5 (BEND) and 8 (REVBY). Use here is made of 40 revolved triangular element connecting 32 nodes leading to 53 degrees of freedom. Results shown in the figure are for average stresses at the intersection of the diagonals of each bay. Again comparison with the results from Ref. 4 are good.



FINITE ELEMENT IDEALIZATION USING REVOLVE TRIANGULAR ELEMENTS



REVBY Sample Problem #4 Thermoplastic Analysis of a Circular Disk

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

1.0E 00 1 12
 1.1E 00 2 13
 1.2E 00 3 14
 1.3E 00 4 15
 1.4E 00 5 16
 1.5E 00 6 17
 1.6E 00 7 18
 1.7E 00 8 19
 1.8E 00 9 20
 1.9E 00 10 21
 2.0E 00 11 22
 1.05E 00 23
 1.15E 00 24
 1.25E 00 25
 1.35E 00 26
 1.45E 00 27
 1.55E 00 28
 1.65E 00 29
 1.75E 00 30
 1.85E 00 31
 1.95E 00 32

BLANK CARD

00.0E 00 1 -11
 0.10E 00 12 -22
 0.05E 00 23 -32

BLANK CARD

0.0E 00 1 -32

BLANK CARD

100000 1 -11
 110000 12 -32

BLANK CARD

BLANK CARD

1.CCCCC000E 07 1.00000000E 07 1.00000000E 07 0.30000000E 00 0.30000000E 00
 0.3CCCC000E 00 3.8461530E 06 3.8461530E 06 3.8461530E 06 0.00000000E 00
 2.CCCCC000E 03 2.00000000E 03 2.00000000E 03 1.1547004E 03 5.00000000E 00
 4.CCCCC000E 03 2.00000000E 03 1.00000000E 07 1.1547004E 03 1.1547004E 03
 0.1CCCC000E-04 0.10000000E-04 0.10000000E-04
 1 -40

BLANK CARD

BLANK CARD

BLANK CARD

C.00000000E 00
 1.00000000E 00 0.0L00 0.0E00
 1 12

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

23	9.2961067E-01	0.0E00	0.0E00
13	8.6249648E-01	0.0E00	0.0E00
24	7.9836614E-01	0.0E00	0.0E00
14	7.3696559E-01	0.0E00	0.0E00
25	6.7807190E-01	0.0E00	0.0E00
15	6.2148838E-01	0.0E00	0.0E00
26	5.6704059E-01	0.0E00	0.0E00
16	5.1457317E-01	0.0E00	0.0E00
27	4.6394710E-01	0.0E00	0.0E00
17	4.1503750E-01	0.0E00	0.0E00
28	3.6773178E-01	0.0E00	0.0E00
7	3.2192809E-01	0.0E00	0.0E00
29	2.7753398E-01	0.0E00	0.0E00
8	2.3446525E-01	0.0E00	0.0E00
30	1.9264508E-01	0.0E00	0.0E00
9	1.5200309E-01	0.0E00	0.0E00
31	1.1247473E-01	0.0E00	0.0E00
10	7.4000581E-02	0.0E00	0.0E00
32	3.6525879E-02	0.0E00	0.0E00
11	22	0.0E00	0.0E00

SEND
-40
-32

BLANK CARD

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

BLANK CARD

-40

BLANK CARD

-52

BLANK CARD

2.0000000E C2 0.0100000E 00

BLANK CARD

STOP

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

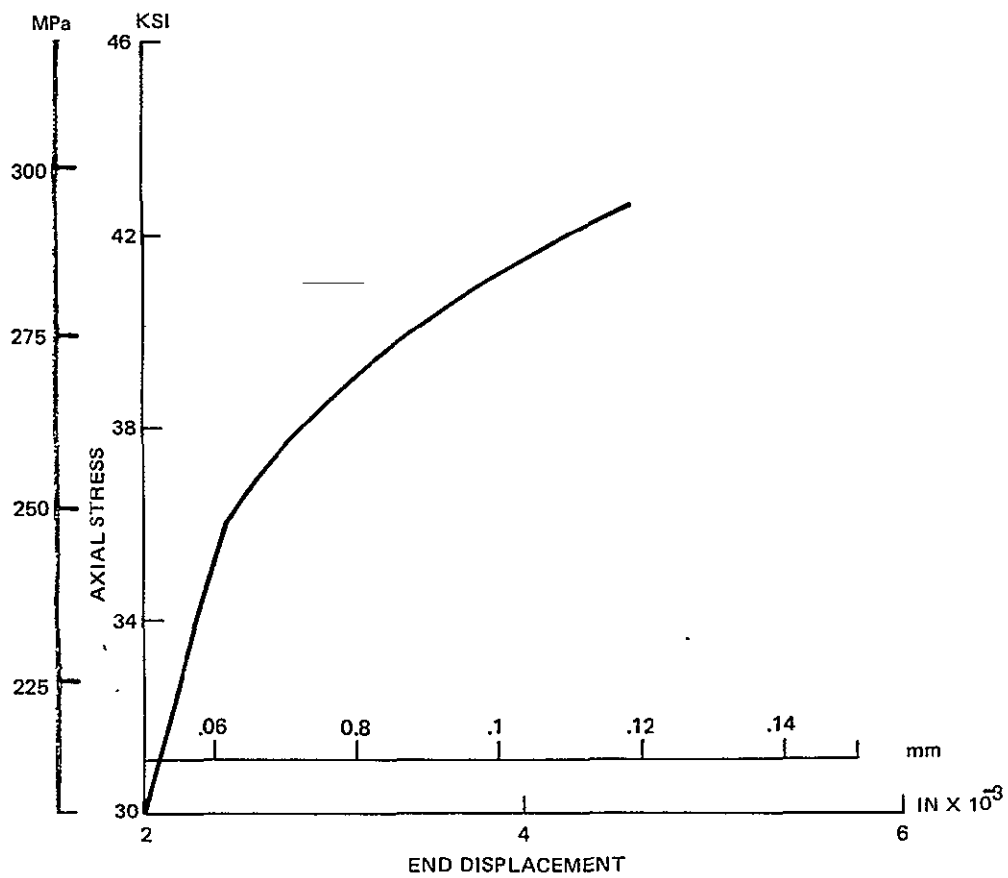
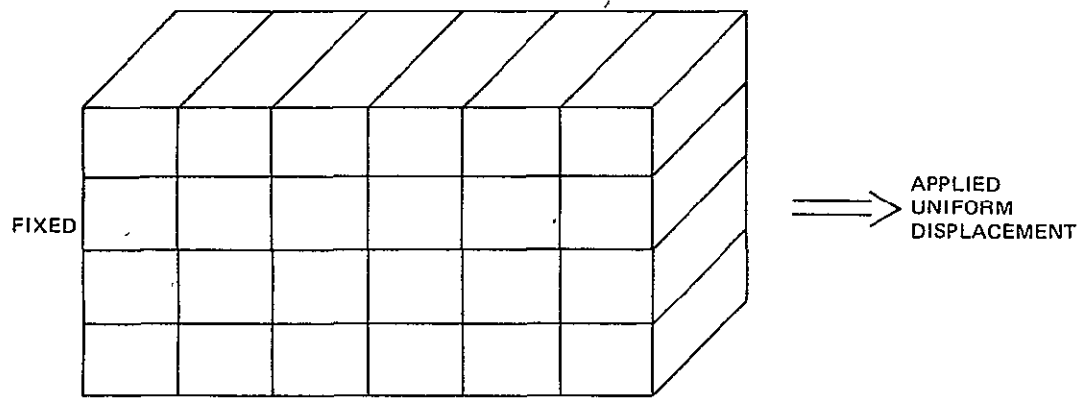
Example Problem No. 11

Program: HEX

Problem Title: Bar Subjected to an End Displacement

Comments: This is a simple statically determinate problem to demonstrate the use of the HEX element. The automatic mesh generation feature is used (MSGN) to specify a 1 x 6 x 4 mesh of 24 elements. An applied edge displacement is imposed on one end.

 The figure shows the axial stress versus applied end displacement.



HEX SAMPLE PROBLEM # 1 BAR SUBJECTED TO AN END DISPLACEMENT

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

HEX SAMPLE PROBLEM # 1 LONG BAR SUBJECTED TO AN APPLIED END DISPLACEMENT

0 5 1 1 5

1 -70

MSGN

1 6 4 1 1

SEND

0.0000000E 00	1	3	5	7	9	9	11	13	15	17	19	21	23
0.0000000E 00	25	27	29	31	33	35	37	39	41	43	45	47	49
0.0000000E 00	51	53	55	57	59	61	63	65	67	69			
2.5000000E-01	2	4	6	8	10	12	14	16	18	20	22	24	26
2.5000000E-01	28	30	32	34	36	38	40	42	44	46	48	50	52
2.5000000E-01	54	56	58	60	62	64	66	68	70				

BLANK CARD

0.0000000E 00	1	-10
5.0000000E-01	11	-20
8.7500000E-01	21	-30
1.2500000E 00	31	-40
1.5000000E 00	41	-50
1.7500000E 00	51	-60
2.0000000E 00	61	-70

BLANK CARD

-5.0000000E-01	1	2	11	12	21	22	31	32	41	42	51	52	61
-5.0000000E-01	62												
-2.5000000E-01	3	4	13	14	23	24	33	34	43	44	53	54	63
-2.5000000E-01	64												
0.0000000E 00	5	6	15	16	25	26	35	36	45	46	55	56	65
0.0000000E 00	66												
2.5000000E-01	7	8	17	18	27	28	37	38	47	48	57	58	67
2.5000000E-01	68												
5.0000000E-01	9	10	19	20	29	30	39	40	49	50	59	60	69
5.0000000E-01	70												

BLANK CARD

001	1	3	7	9
101	2	4	8	10
000	5			
100	6			
121	61	-70		

BLANK CARD

61	2	70	2	1.0
62	2	70	2	1.0
63	2	70	2	1.0
64	2	70	2	1.0
65	2	70	2	1.0
66	2	70	2	1.0

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

67 2 70 2 1.0
68 2 70 2 1.0
69 2 70 2 1.0
70 2 70 2 .003114

BLANK CARD

MAT1 3.0 E 07 3.0 E 07 3.0 E 07 .33333E 00 .33333E 00
.33333E 00 1.125 E 07 1.125 E 07 1.125 E 07
0.0 E 00 0.0 E 00 0.0 E 00
12. E-06 12. E-06 12. E-06
3.6 E 04 3.6 E 04 3.6 E 04 2.0784E 04 2.0784E 04
2.0784 E 04 10. E 00 4.0 E 04 3.6 E 04 3.0 E 07
1 -24

SEND
SEND
-100

BLANK CARD

-100

BLANK CARD

1.5

.01

BLANK CARD

STOP

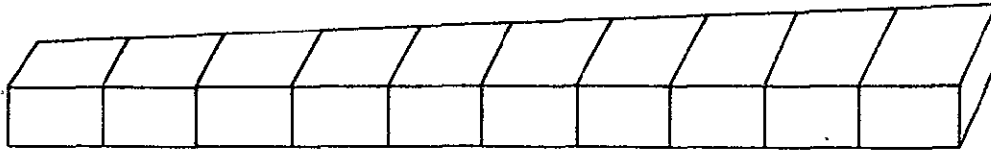
.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

Example Problem No. 12

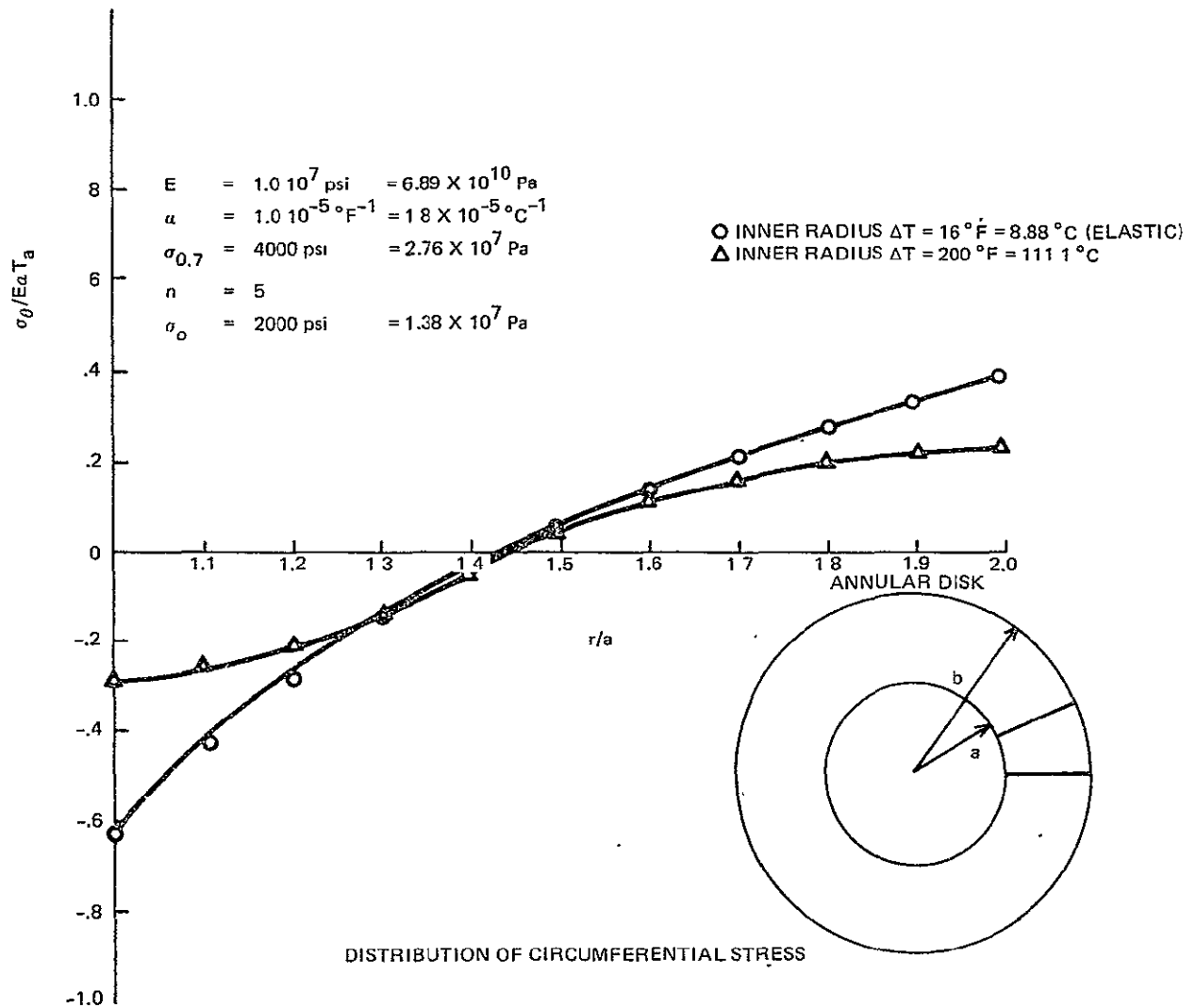
Program: HEX

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem is the same as problems 5 (BEND), 8, and 9 (REVBVY). Here the idealization of a slice of the disk is used with 10 hexahedra elements with midside nodes in the radial direction. Thirty Lobatto stress points are used in each direction, 10 in the radial, 3 in the circumference, and 1 in the thickness direction.



FINITE ELEMENT MODEL OF A SECTOR OF THE DISK



PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

1.0814254 E+00 43 46
1.7961344 E+00 49 52
1.8885910 E+00 55 58
1.9890437 E+00 61 64
.9988205 E+00 2 5
1.1984924 E+00 8 11
1.1983554 E+00 14 17
1.2982100 E+00 20 23
1.3980813 E+00 26 29
1.4979443 E+00 32 35
1.5978072 E+00 38 41
1.6976702 E+00 44 47
1.7975331 E+00 50 53
1.8973961 E+00 56 59
1.9972590 E+00 62 65
0.0 101 -140

BLANK CARD

7-67

0.0 E+00 3 -3 -06
.5233354 E+00 2 5
.2575695 E+00 8 11
.0628031 E+00 14 17
.0680367 E+00 20 23
.0732703 E+00 26 29
.0785039 E+00 32 35
.0837375 E+00 38 41
.0889711 E+00 44 47
.0942047 E+00 50 53
.0994383 E+00 56 59
.1046712 E+00 62 65
.1045204 E+00 1 4
.1149813 E+00 7 10
.1254341 E+00 13 16
.1358870 E+00 19 22
.1463398 E+00 25 28
.1567920 E+00 31 34
.1672455 E+00 37 40
.1776982 E+00 43 46
.1881512 E+00 49 52
.1986040 E+00 55 58
.2090569 E+00 61 64
0.0 101 -140

BLANK CARD

0.0 E+00 3 -6 -03 2 -6 -02 1 -6 -01
0.1 E+00 4 -6 -04 5 -6 -05 6 -6 -06

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

0.0

101 -140

BLANK CARD

100	3	-6	-63	1	-6	-61
100	101	-4	-137	103	-4	-139
110	2	-6	-62			
101	4	-6	-64	6	-6	-66
101	102	-4	-136	104	-4	-140

BLANK CARD

1	1	1	1	.9945218	1	2	-.1045284
1	2	1	1	.1045284	1	2	.9945218
4	1	4	1	.9945218	4	2	-.1045284
4	2	4	1	.1045284	4	2	.9945218
7	1	7	1	.9945218	7	2	-.1045284
7	2	7	1	.1045284	7	2	.9945218
10	1	10	1	.9945218	10	2	-.1045284
10	2	10	1	.1045284	10	2	.9945218
13	1	13	1	.9945218	13	2	-.1045284
13	2	13	1	.1045284	13	2	.9945218
16	1	16	1	.9945218	16	2	-.1045284
16	2	16	1	.1045284	16	2	.9945218
19	1	19	1	.9945218	19	2	-.1045284
19	2	19	1	.1045284	19	2	.9945218
22	1	22	1	.9945218	22	2	-.1045284
22	2	22	1	.1045284	22	2	.9945218
25	1	25	1	.9945218	25	2	-.1045284
25	2	25	1	.1045284	25	2	.9945218
28	1	28	1	.9945218	28	2	-.1045284
28	2	28	1	.1045284	28	2	.9945218
31	1	31	1	.9945218	31	2	-.1045284
31	2	31	1	.1045284	31	2	.9945218
34	1	34	1	.9945218	34	2	-.1045284
34	2	34	1	.1045284	34	2	.9945218
37	1	37	1	.9945218	37	2	-.1045284
37	2	37	1	.1045284	37	2	.9945218
40	1	40	1	.9945218	40	2	-.1045284
40	2	40	1	.1045284	40	2	.9945218
43	1	43	1	.9945218	43	2	-.1045284
43	2	43	1	.1045284	43	2	.9945218
46	1	46	1	.9945218	46	2	-.1045284
46	2	46	1	.1045284	46	2	.9945218
49	1	49	1	.9945218	49	2	-.1045284
49	2	49	1	.1045284	49	2	.9945218
52	1	52	1	.9945218	52	2	-.1045284
52	2	52	1	.1045284	52	2	.9945218

7-68

.....1.....2.....3.....4.....5.....6.....7.....8
 123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

55	1	55	1	.9945218	55	2	-.1045284
55	2	55	1	.1045284	55	2	.9945218
58	1	58	1	.9945218	58	2	-.1045284
58	2	58	1	.1045284	58	2	.9945218
61	1	61	1	.9945218	61	2	-.1045284
61	2	61	1	.1045284	61	2	.9945218
64	1	64	1	.9945218	64	2	-.1045284
64	2	64	1	.1045284	64	2	.9945218
101	1	101	1	.9945218	101	2	-.1045284
101	2	101	1	.1045284	101	2	.9945218
105	1	105	1	.9945218	105	2	-.1045284
105	2	105	1	.1045284	105	2	.9945218
109	1	109	1	.9945218	109	2	-.1045284
109	2	109	1	.1045284	109	2	.9945218
113	1	113	1	.9945218	113	2	-.1045284
113	2	113	1	.1045284	113	2	.9945218
117	1	117	1	.9945218	117	2	-.1045284
117	2	117	1	.1045284	117	2	.9945218
121	1	121	1	.9945218	121	2	-.1045284
121	2	121	1	.1045284	121	2	.9945218
125	1	125	1	.9945218	125	2	-.1045284
125	2	125	1	.1045284	125	2	.9945218
129	1	129	1	.9945218	129	2	-.1045284
129	2	129	1	.1045284	129	2	.9945218
133	1	133	1	.9945218	133	2	-.1045284
133	2	133	1	.1045284	133	2	.9945218
137	1	137	1	.9945218	137	2	-.1045284
137	2	137	1	.1045284	137	2	.9945218
102	1	102	1	.9945218	102	2	-.1045284
102	2	102	1	.1045284	102	2	.9945218
106	1	106	1	.9945218	106	2	-.1045284
106	2	106	1	.1045284	106	2	.9945218
110	1	110	1	.9945218	110	2	-.1045284
110	2	110	1	.1045284	110	2	.9945218
114	1	114	1	.9945218	114	2	-.1045284
114	2	114	1	.1045284	114	2	.9945218
118	1	118	1	.9945218	118	2	-.1045284
118	2	118	1	.1045284	118	2	.9945218
122	1	122	1	.9945218	122	2	-.1045284
122	2	122	1	.1045284	122	2	.9945218
126	1	126	1	.9945218	126	2	-.1045284
126	2	126	1	.1045284	126	2	.9945218
130	1	130	1	.9945218	130	2	-.1045284
130	2	130	1	.1045284	130	2	.9945218

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

134	1	134	1	.9945218	134	2	-.1045284
134	2	134	1	.1045284	134	2	.9945218
138	1	138	1	.9945218	138	2	-.1045284
138	2	138	1	.1045284	138	2	.9945218

BLANK CARD

MA11	1.0	E+07	1.0	E+07	1.0	E+07	.3	.3
.5		3.846153	E+06	3.846153	E+06	3.846153	E+06	
.1	E-04	.1	E-04	.1	E-04			
2.0	E+03	2.0	E+03	2.0	E+03	1.1547004	E+03	1.1547004 E+03
1.1547004	E+03	5.0	E+06	4.0	E+03	2.0	E+03	1.0 E+07

LOBA7		10	1	3
1	-10			

SENO

FLMP 0.0

1.0	1	-0
.9296107	101	-104
.8624984	7	-12
.7983861	105	-108
.7369655	13	-18
.6766719	109	-112
.6214833	19	-24
.5670406	113	-116
.5145732	25	-36
.4639471	117	-120
.4158375	31	-36
.3677318	121	-124
.3219251	37	-42
.2775340	125	-128
.2344052	43	-48
.1926451	129	-132
.1520030	49	-54
.1124747	133	-136
.0746054	55	-60
.03852588	137	-140
.0	61	-66

BLANK CARD

SEGL 1 -10

BLANK CARD

1 -60 101 -140

BLANK CARD

200.0 .05

7-70

.....1.....2.....3.....4.....5.....6.....7.....8
 1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

BLANK CARD

STOP

7-71

.....1.....2.....3.....4.....5.....6.....7.....8
123456789012345678901234567890123456789012345678901234567890

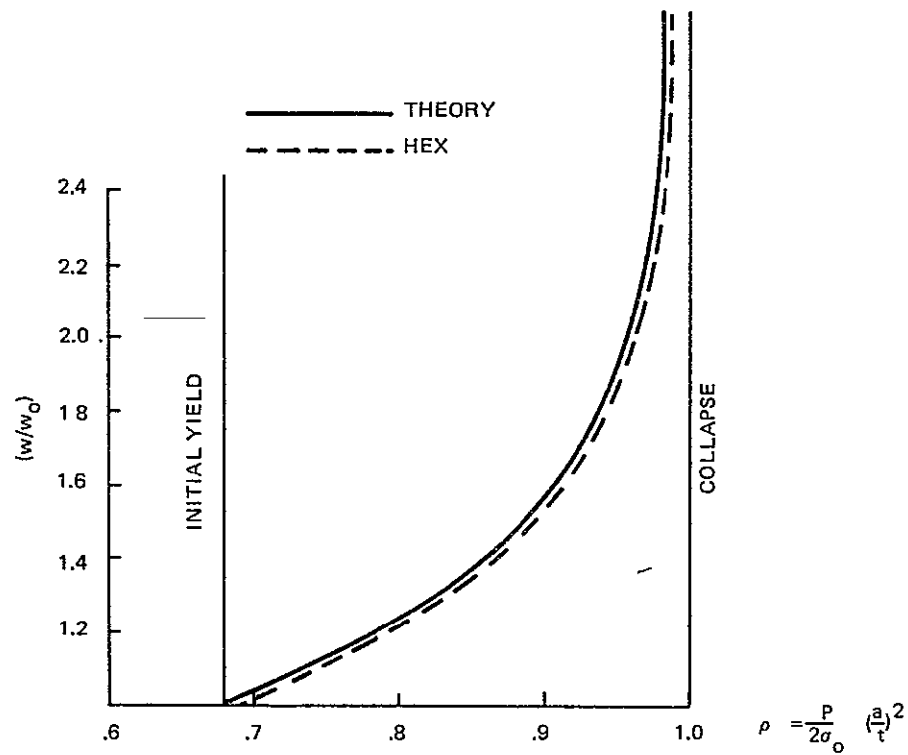
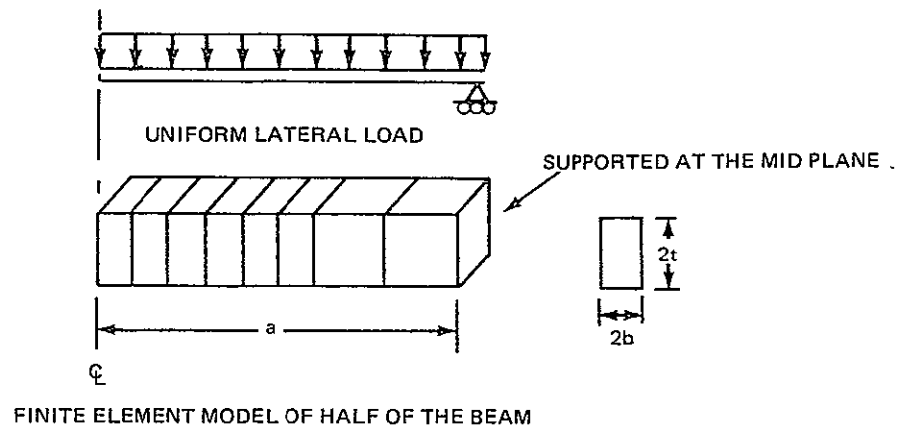
Example Problem No. 13

Program: HEX

Problem Title: Collapse of a Simply Supported Uniformly Loaded Beam

Comments: Ten 20 node hexahedra elements are used to model half of a simply supported beam. A $1 \times 1 \times 8$ array of Lobatto points are used (8 through the thickness) to determine stresses within each element. Lobatto points were chosen in order to have stress points at the surface. Eight points were taken through the thickness in order to accurately define an elastic plastic boundary through the thickness.

The figure shows the central deflection versus load for an ideally plastic material. Results are in good agreement with Ref. 5.



LOAD VS. NON-DIMENSIONAL CENTER DEFLECTION (w_0 CORRESPONDS TO DISPLACEMENT AT INITIAL YIELD) OF A SIMPLY-SUPPORTED BEAM. HEX RESULTS CORRESPOND TO AN1X1X8 ARRAY OF LOBATTO POINTS.

HEX Sample Problem #3 Collapse of a Uniformly Loaded Beam

74-7

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

	0.5	111	112												
	0.6	113	114												
	0.8	115	116												
BLANK CARD	1.0	117	118												
	0.0	1	-86												
	0.1	4	5	6	9	10	14	15	16	19	20	24	25	26	
		29	30	34	35	36	39	40	44	45	46	49	50	54	
		55	56	59	60	64	65	66	69	70	74	75	76	79	
		80	84	85	86										
BLANK CARD	0.05	101	-118												
	0.0	1	-86												
	0.05	2	5	12	15	22	25	32	35	42	45	52	55	62	
		65	72	75	82	85									
7-75	0.1	3	6	8	10	13	16	18	20	23	26	28	30	33	
		36	38	40	43	46	48	50	53	56	58	60	63	66	
		68	70	73	76	78	80	83	86						
	0.0	101	-2	-117											
BLANK CARD	0.1	102	-2	-118											
	0.11	1	-6												
	110	82	85												
	001	101	102												
BLANK CARD	101	103	-118												
BLANK CARD															
	MAT1	3.0E+07	3.0E+07	3.0E+07	0.3										
	0.3	1.15385E+07	1.15385E+07	1.15385E+07	0.3										
BLANK CARD															
BLANK CARD															
		30000.0	30000.0	30000.0	17320.0										
		17320.0			30000.0										
	1	-8													
LOPAT		1			1					8					
1	-8														
SEND															
SURF															
5		3	13	16	6	8	104	10	102						
3							-1.0								
6							-1.0								
8							-1.0								
10							-1.0								
13							-1.0								

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

PROGRAM LISTING OF INPUT DATA CARDS

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

16							-1.0		
102							-1.0		
104							-1.0		
8	13	23	26	16	16	106	20	104	
13							-1.0		
16							-1.0		
18							-1.0		
20							-1.0		
23							-1.0		
26							-1.0		
104							-1.0		
106							-1.0		
8	23	33	36	26	28	108	30	106	
23							-1.0		
26							-1.0		
28							-1.0		
30							-1.0		
33							-1.0		
36							-1.0		
106							-1.0		
108							-1.0		
8	33	43	46	36	38	110	40	108	
33							-1.0		
36							-1.0		
38							-1.0		
40							-1.0		
43							-1.0		
46							-1.0		
108							-1.0		
110							-1.0		
8	43	53	56	46	48	112	50	110	
43							-1.0		
46							-1.0		
48							-1.0		
50							-1.0		
53							-1.0		
56							-1.0		
110							-1.0		
112							-1.0		
8	53	63	66	56	58	114	60	112	
53							-1.0		
56							-1.0		
58							-1.0		
60							-1.0		

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

```

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

```

STOP

.....1.....2.....3.....4.....5.....6.....7.....8
1234567890123456789012345678901234567890123456789012345678901234567890

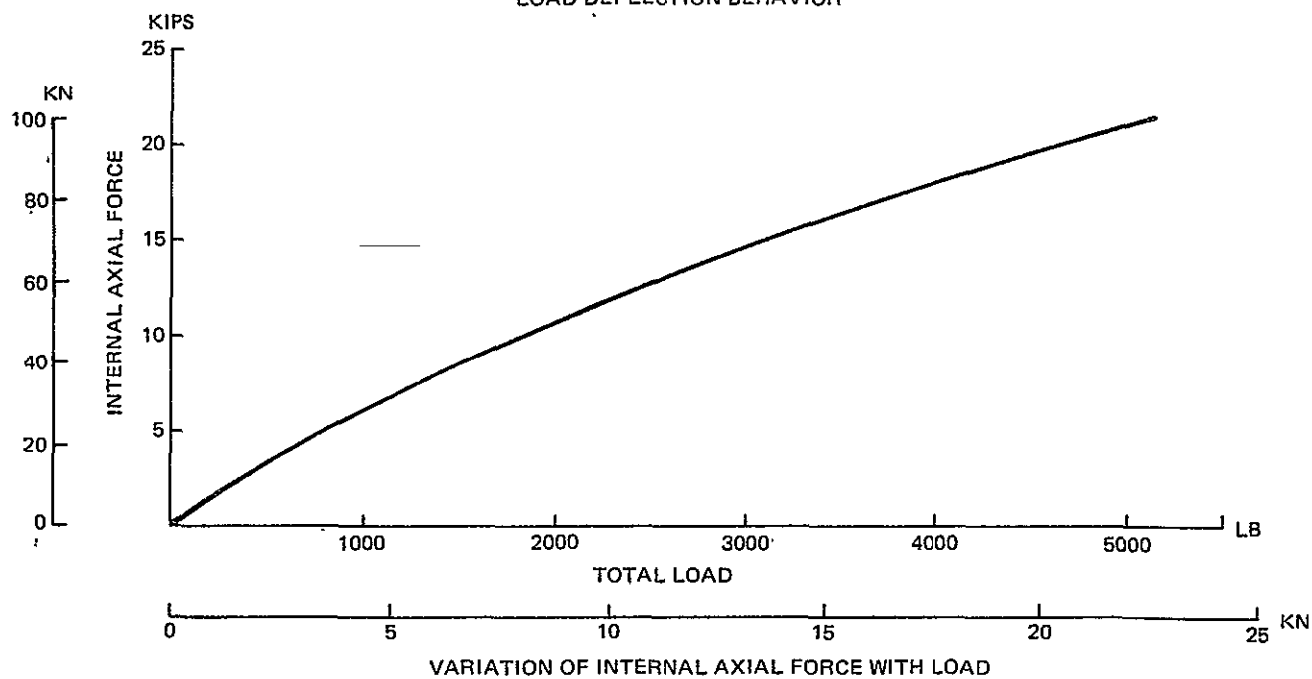
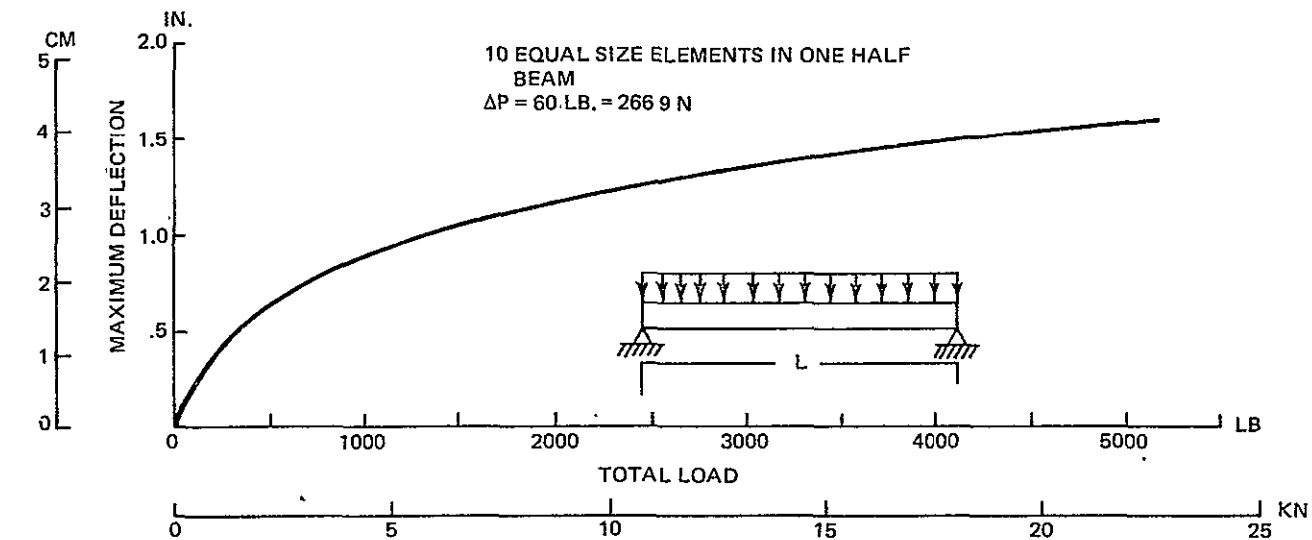
Example Problem No. 14

Program: OPLANE-GM

Problem Title: Uniformly Loaded Restrained Beam

Comments: Ten beam elements are used to model half a simply supported restrained beam. A full tangent modulus method is used (PTAN = 0.0).

The figure shows the central deflection and internal axial force versus the total load.



OPLANE-MG Sample Problem # 1 Uniformly Loaded Restrained Beam

PROGRAM LISTING OF INPUT DATA CARDS

1234567890¹1234567890²1234567890³1234567890⁴1234567890⁵1234567890⁶1234567890⁷1234567890⁸

UPLANENG SAMPLE PROBLEM = 1 UNIFORMLY LOADED RESTRAINED BEAM

1	5	1	0	1	10		
100.0			100.0		0.0		1.0
1	-12						

BEAM	1	1	2	12
BEAM	2	2	3	12
BEAM	3	3	4	12
BEAM	4	4	5	12
BEAM	5	5	6	12
BEAM	6	6	7	12
BEAM	7	7	8	12
BEAM	8	8	9	12
BEAM	9	9	10	12
BEAM	10	10	11	12
SEND				

0.0	FF8000	1
2.0	FF8000	2
6.0	FF8000	3
9.0	FF8000	4
12.0	FF8000	5
15.0	FF8000	6
18.0	FF8000	7
21.0	FF8000	8
24.0	FF8000	9
27.0	FF8000	10
30.0	FF8000	11

BLANK CARD C.C. E8CC 1 -12

BLANK CARD C.C. E8CC 1 -11

LC.C E&CC 12

BLANK CARD

CGOOGC	12	
CGOOLC	1	
LCLOLC	2	-1C
GOLOGC	11	

BLANK CARD 001000 11

BLANK CARD

MBM 1.0 E&C7 .25
C.C
1 C.C 10
SR EC 1.2
C.C
1.0 C.C
1.2

3.0 ELOS

0.0

1234567890¹1234567890²1234567890³1234567890⁴1234567890⁵1234567890⁶1234567890⁷1234567890⁸

PROGRAM LISTING OF INPUT DATA CARDS

1234567890¹1234567890²1234567890³1234567890⁴1234567890⁵1234567890⁶1234567890⁷1234567890⁸

SEND 1 -10

BMLO

-1.0
1 -10

-1.0

BLANK CARD

SEND

1 -5

BLANK CARD

1 -11

BLANK CARD

BLANK CARD

END

7-82

1234567890¹1234567890²1234567890³1234567890⁴1234567890⁵1234567890⁶1234567890⁷1234567890⁸

REFERENCES

1. Isakson, G., Armen, H. Jr., and Pifko, A., "Discrete-Element Methods for the Plastic Analysis of Structures," NASA Contractor Report CR-803, October 1967.
2. Armen, H. Jr., Pifko, A., and Levine, H., "Finite Element Analysis of Structures in the Plastic Range," NASA Contractor Report CR-1649, February 1971.
3. Armen, H. Jr., Levine, H., Pifko, A., and Levy, A., "Non-linear Analysis of Structures," NASA Contractor Report CR-2351, March 1974.
4. Dastidar, D. and Ghosh, P., "Stresses and Strains in the Plastic Range in an Annular Disk due to Steady State Radial Temperature Variation," International Journal of Mechanical Sciences, Vol. 14, pp. 501-510, 1972.
5. Prager, W. and Hodge, P. Jr., Theory of Perfectly Plastic Solids, J. Wiley and Sons, Inc., 1951.